

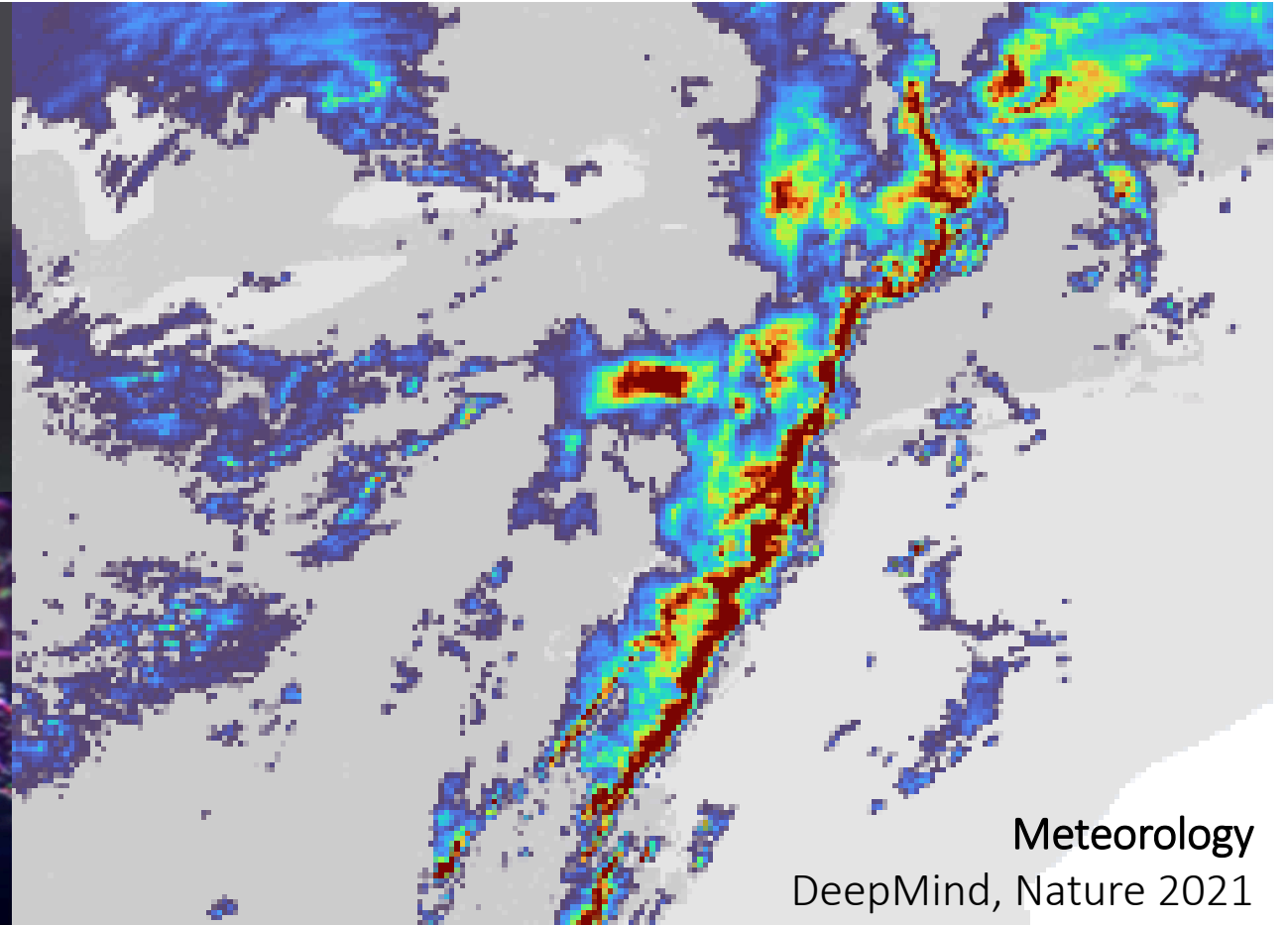


# The Power of Gradients in Inverse Dynamics Problems

Tao Du

MIT CSAIL

# What is a dynamic system?



# What is a dynamic system?

*“A dynamical system is particle or ensemble of particles whose state varies over time and thus obeys differential equations involving time derivatives.”*

---Nature Portfolio

# What is a dynamic system?

States      Time derivatives

$$s, \quad \frac{ds}{dt}, \frac{d^2s}{dt^2}, \dots$$

# What is a dynamic system?

Dynamic model

$$F\left(s, \frac{ds}{dt}, \frac{d^2s}{dt^2}, \dots\right) = 0$$

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Dynamic model

$$F\left(s, \frac{ds}{dt}, \frac{d^2s}{dt^2}, \dots\right) = 0$$

Example

Rigid-body systems: Euler-Lagrange equation

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} - Q = 0$$

Deformable objects: continuum mechanics

$$\nabla \cdot \sigma + f = 0$$

Fluid systems: Navier-Stokes equation

$$\frac{du}{dt} + (u \cdot \nabla)u - \nu \nabla^2 u = -\frac{1}{\rho} \nabla p + g$$

# The input and the output

Input

Parameters

Dynamic model

$$F\left(s, \frac{ds}{dt}, \frac{d^2s}{dt^2}, \dots\right) = 0$$

Example

Rigid-body systems: Euler-Lagrange equation

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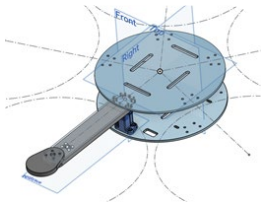
# The input and the output

Input

Parameters

Example

Intrinsic parameters



Extrinsic parameters



Dynamic model

$$F\left(s, \frac{ds}{dt}, \frac{d^2s}{dt^2}, \dots\right) = 0$$

Example

Rigid-body systems: Euler-Lagrange equation

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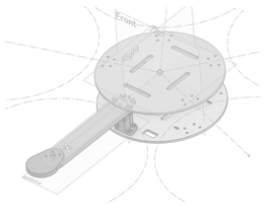
# The input and the output

Input

Parameters

Example

Intrinsic parameters



Extrinsic parameters



Dynamic model

$$F\left(s, \frac{ds}{dt}, \frac{d^2s}{dt^2}, \dots\right) = 0$$

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Rigid-body systems: Euler-Lagrange equation

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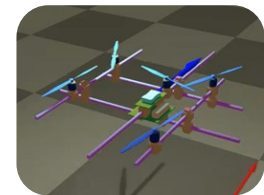
$$\frac{du}{dt} + (u \cdot \nabla)u - \nu \nabla^2 u = -\frac{1}{\rho} \nabla p + g$$

Output

State sequences

Example

States from simulation



States from experiments



# The forward dynamics problem

Given the model and parameters of a dynamic system, compute its state sequence.

# The forward dynamics problem

Given the model and parameters of a dynamic system, compute its state sequence.

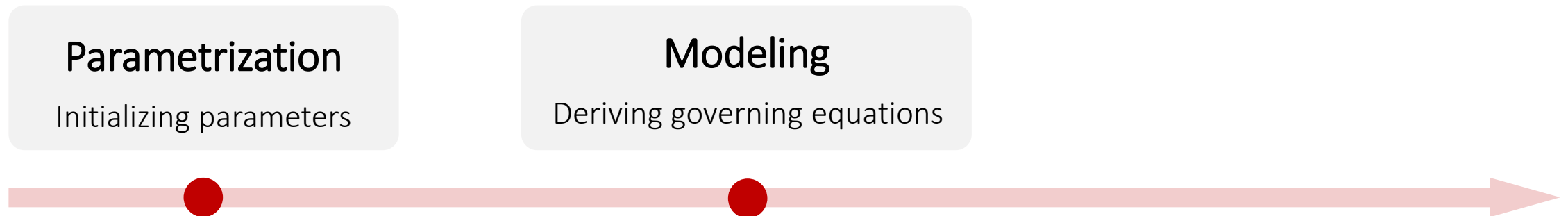
**Parametrization**

Initializing parameters



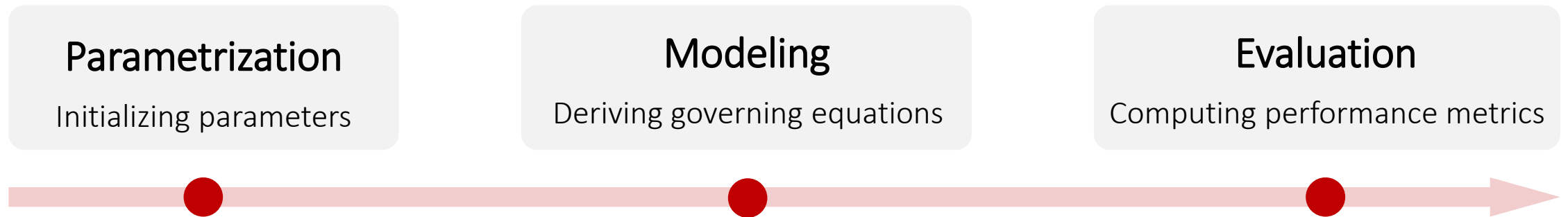
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Given the model and parameters of a dynamic system, compute its state sequence.

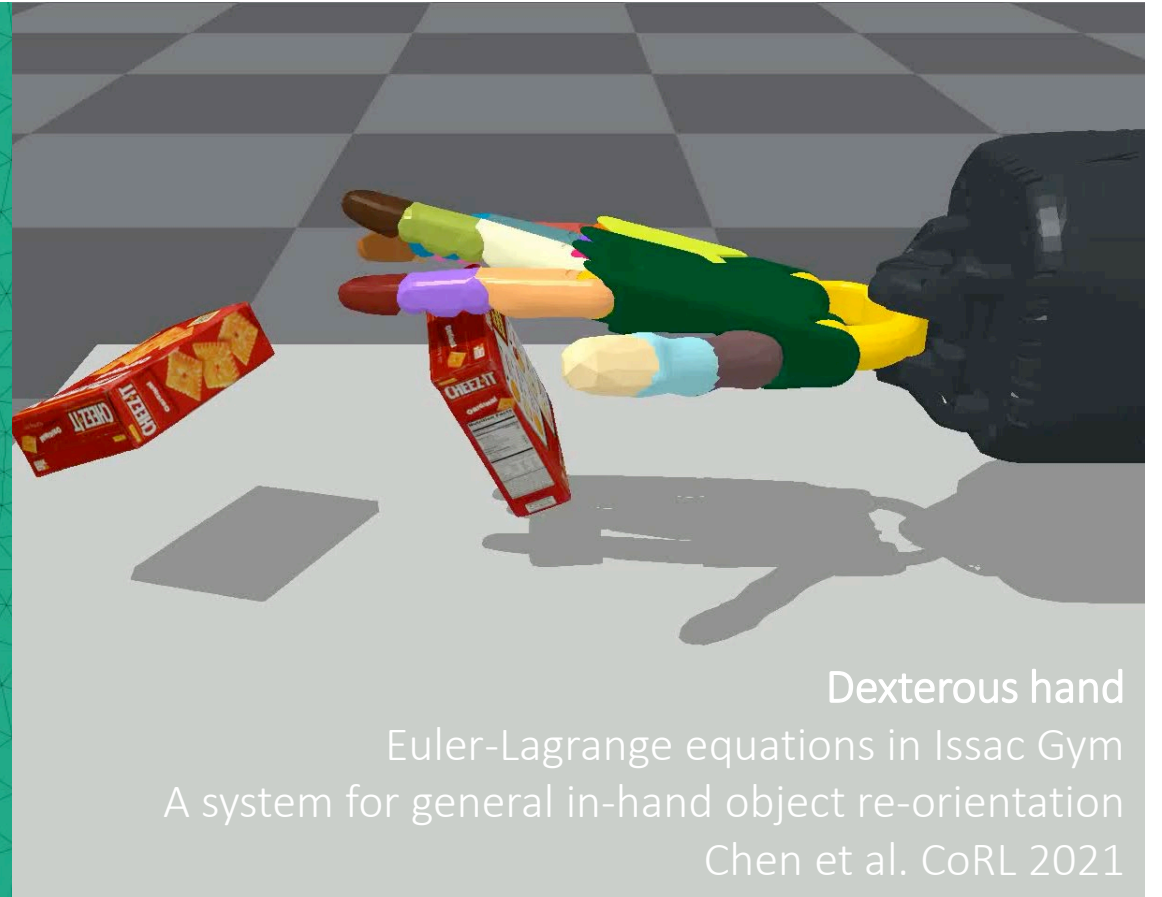
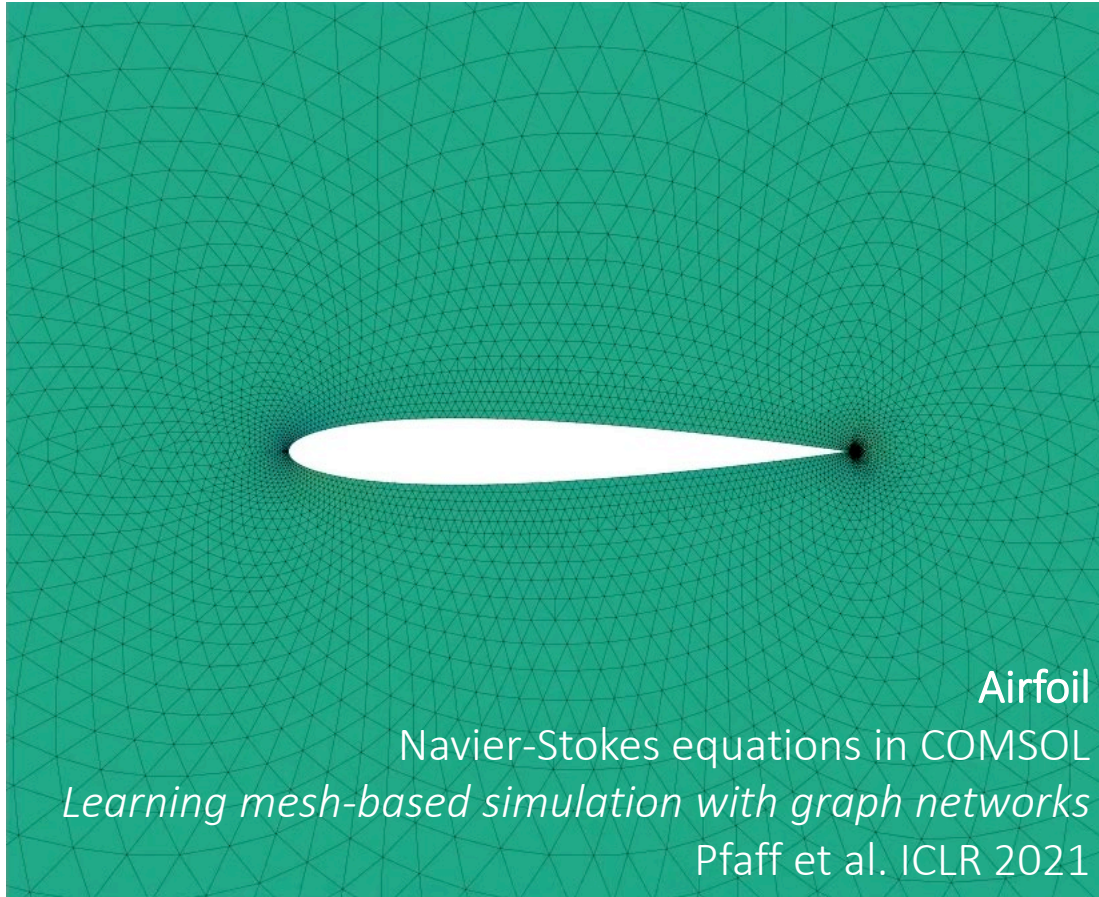


# The forward dynamics problem

Given the model and parameters of a dynamic system, compute its state sequence.



# The forward dynamics problem

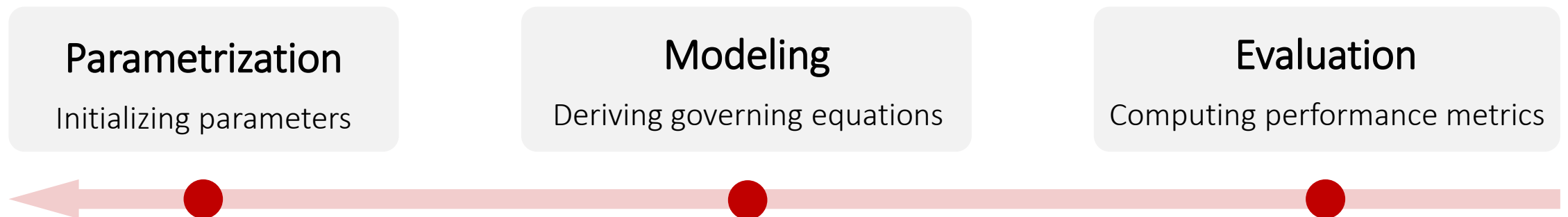


# The inverse dynamics problem

Given the state sequence of a dynamic system, infer its model and parameters.

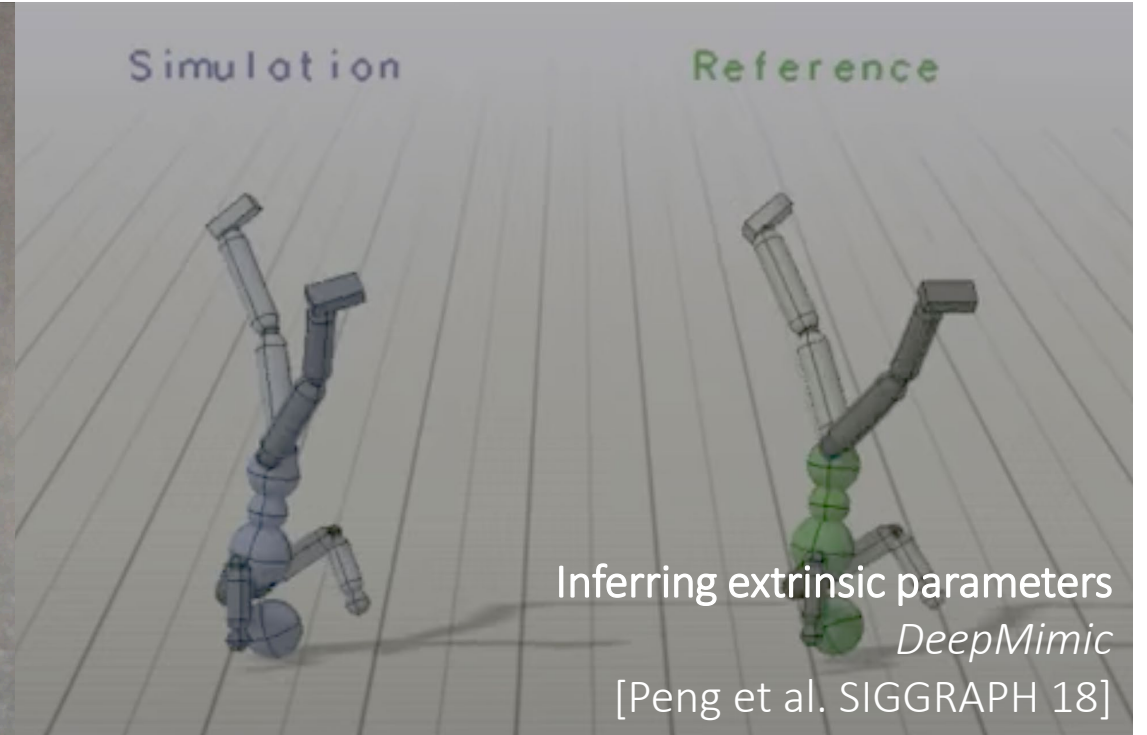
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Given the state sequence of a dynamic system, infer its model and parameters.

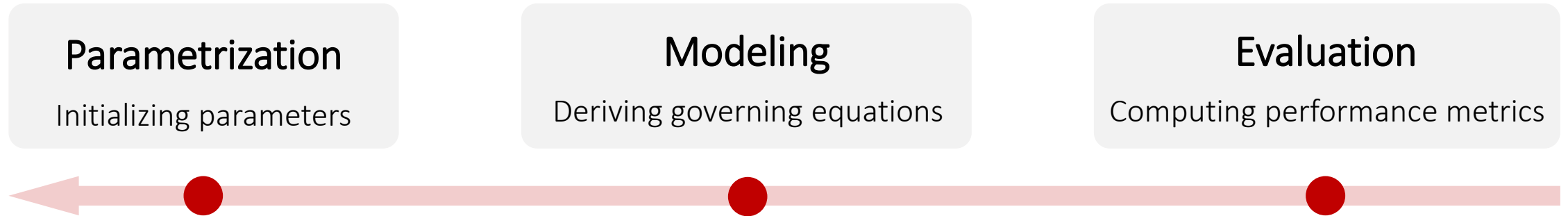




# The inverse dynamics problem



# Our topic today: the gradient methodology



# Our topic today: the gradient methodology

## ▽ Parametrization

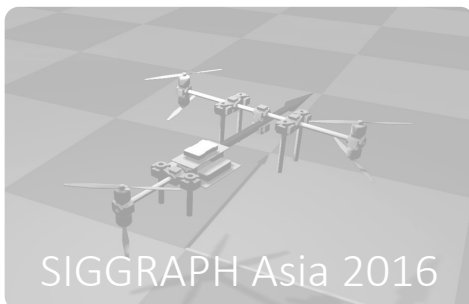
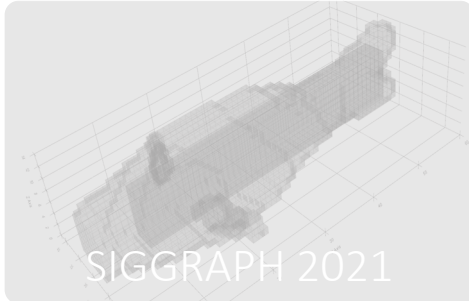
Initializing parameters

## Modeling

Deriving governing equations

## Evaluation

Computing performance metrics



# Our topic today: the gradient methodology

## ▽ Parametrization

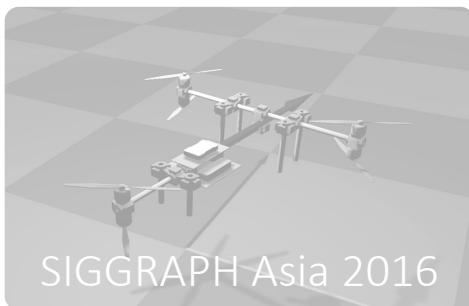
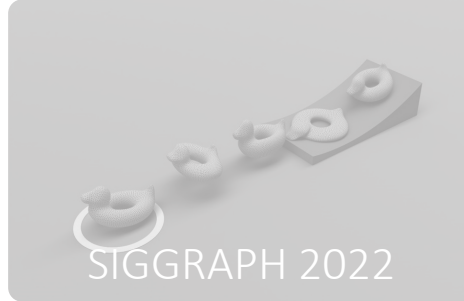
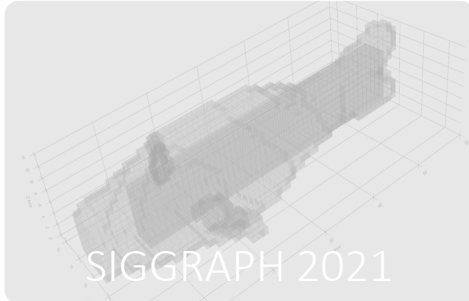
Initializing parameters

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Deriving governing equations

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# Our topic today: the gradient methodology

## ▽ Parametrization

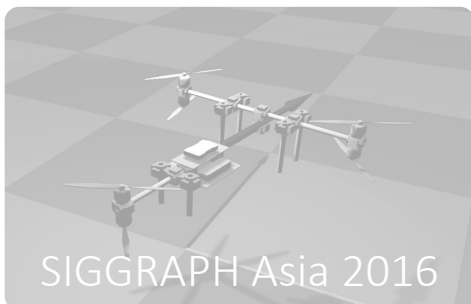
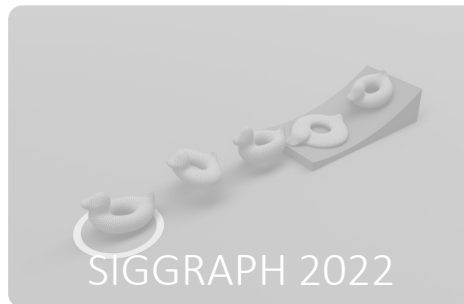
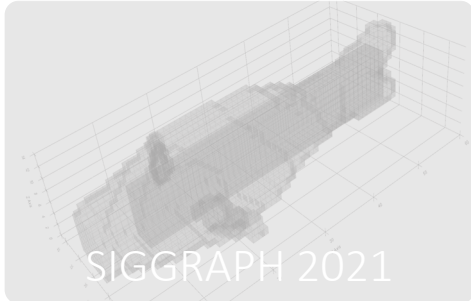
Initializing parameters

## ▽ Modeling

Deriving governing equations

## ▽ Evaluation

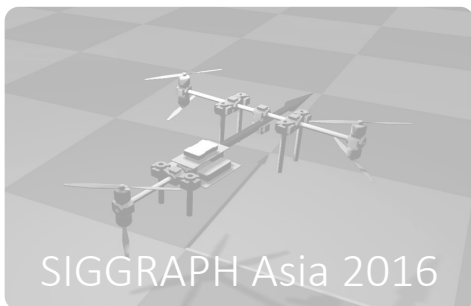
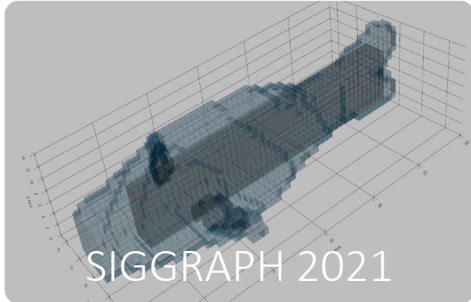
Computing performance metrics



# Our topic today: the gradient methodology

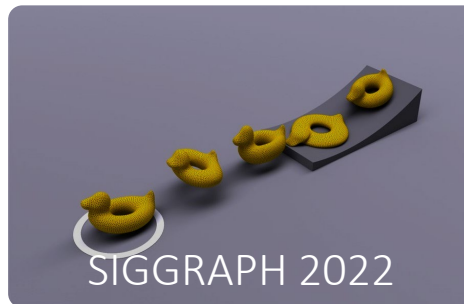
## ▽ Parametrization

Initializing parameters



## ▽ Modeling

Deriving governing equations

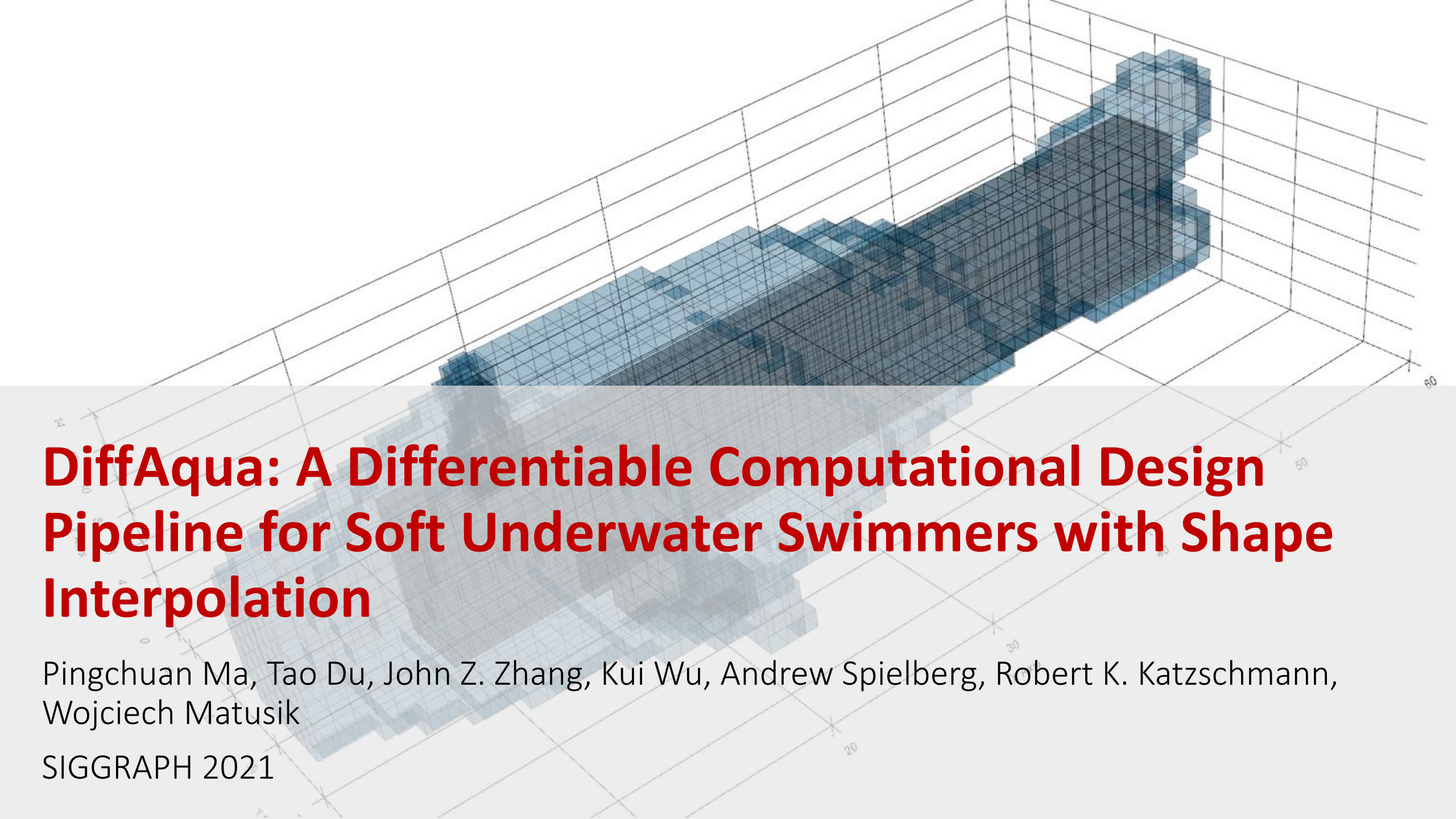


## ▽ Evaluation

Computing performance metrics







# DiffAqua: A Differentiable Computational Design Pipeline for Soft Underwater Swimmers with Shape Interpolation

Pingchuan Ma, Tao Du, John Z. Zhang, Kui Wu, Andrew Spielberg, Robert K. Katzschmann, Wojciech Matusik

SIGGRAPH 2021

# Problem statement

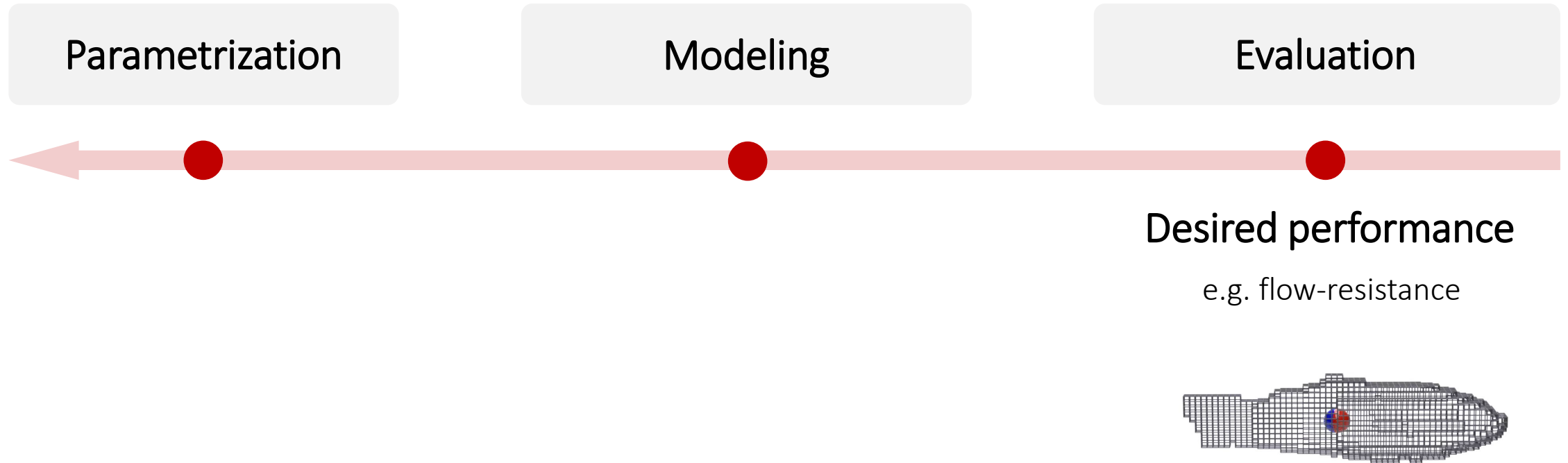
Find the optimal *shape* and *control* of soft robotic fishes to achieve *extremal* performance for underwater tasks.



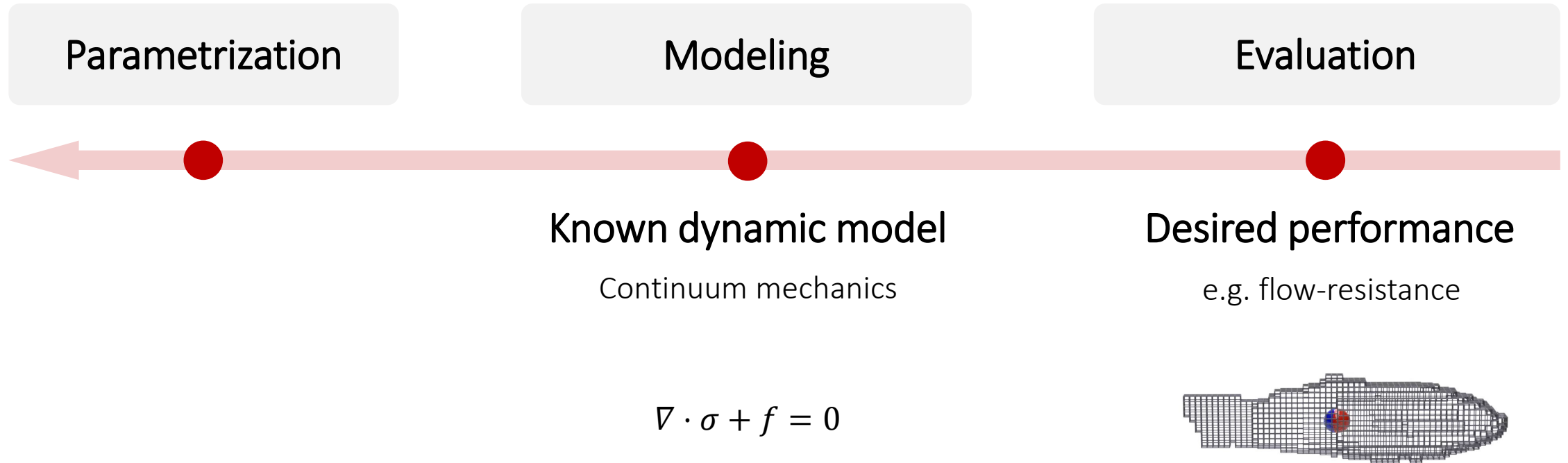
# Applications of soft robotic fish



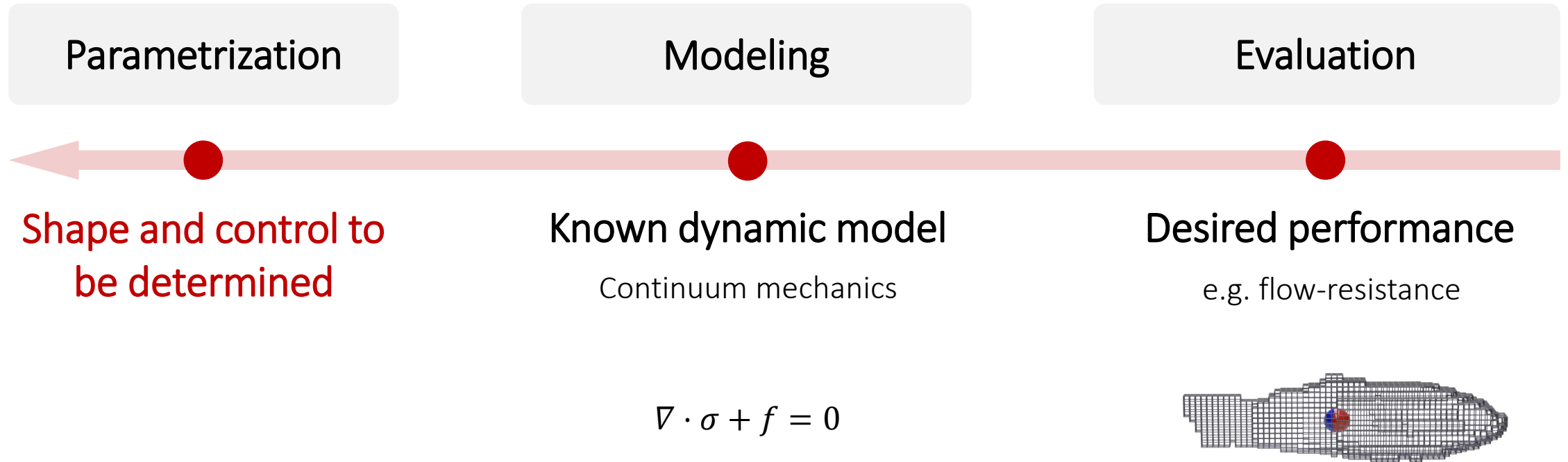
# Why is it an inverse dynamics problem



# Why is it an inverse dynamics problem



# Why is it an inverse dynamics problem



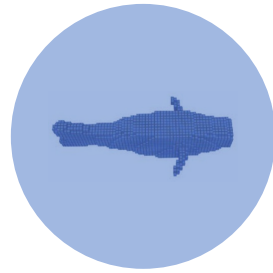
# The challenges

Fishes are **soft**: many degrees of freedom are needed.

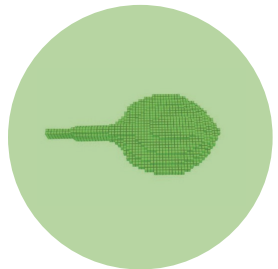
Fishes are **diverse**: it's difficult to find one compact representation for all.



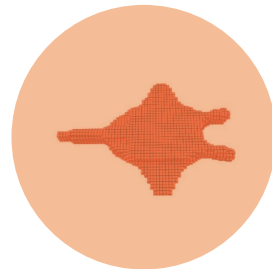
# Parametrization is the key



40k DoFs  
3 fins

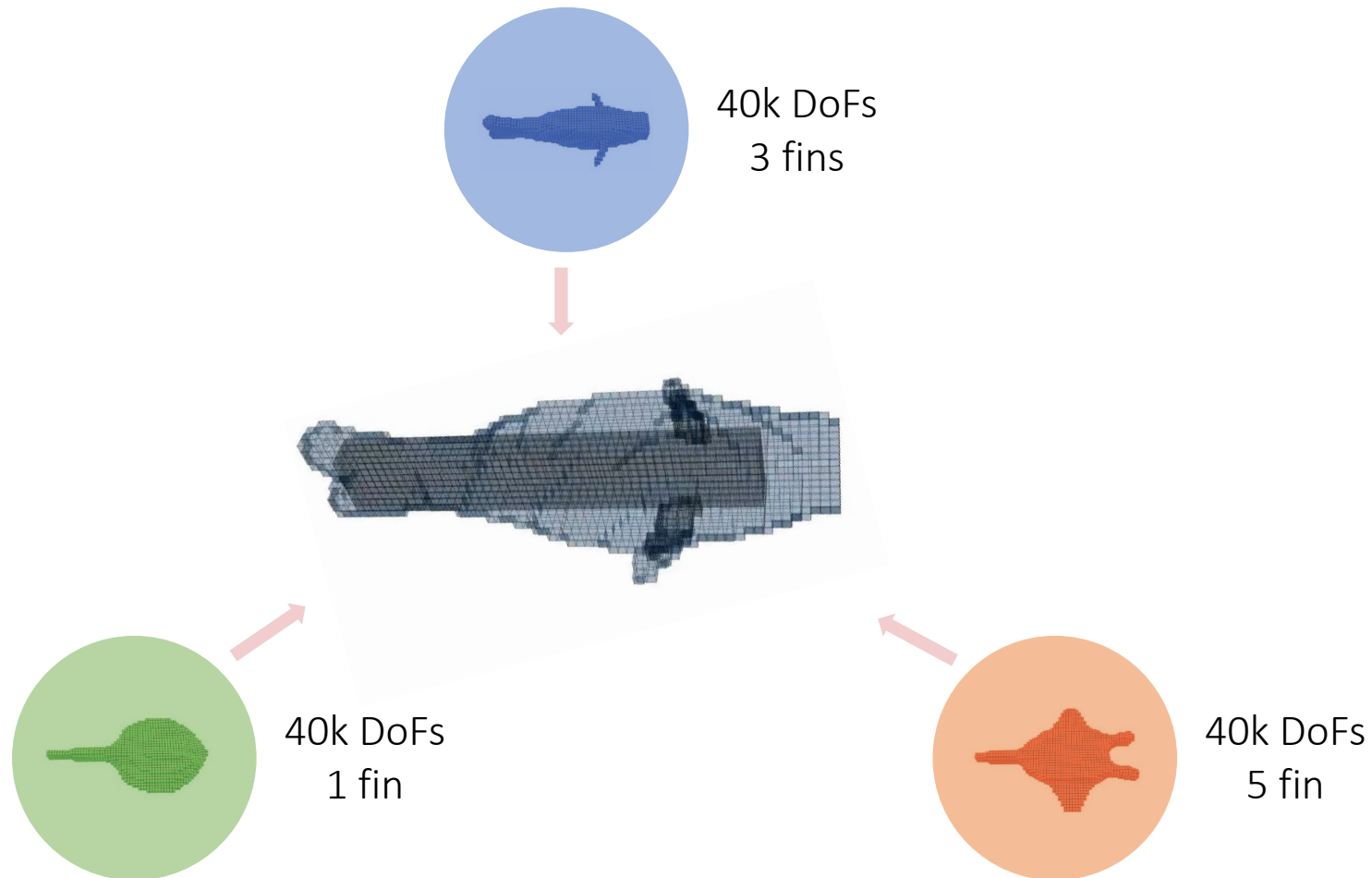


40k DoFs  
1 fin

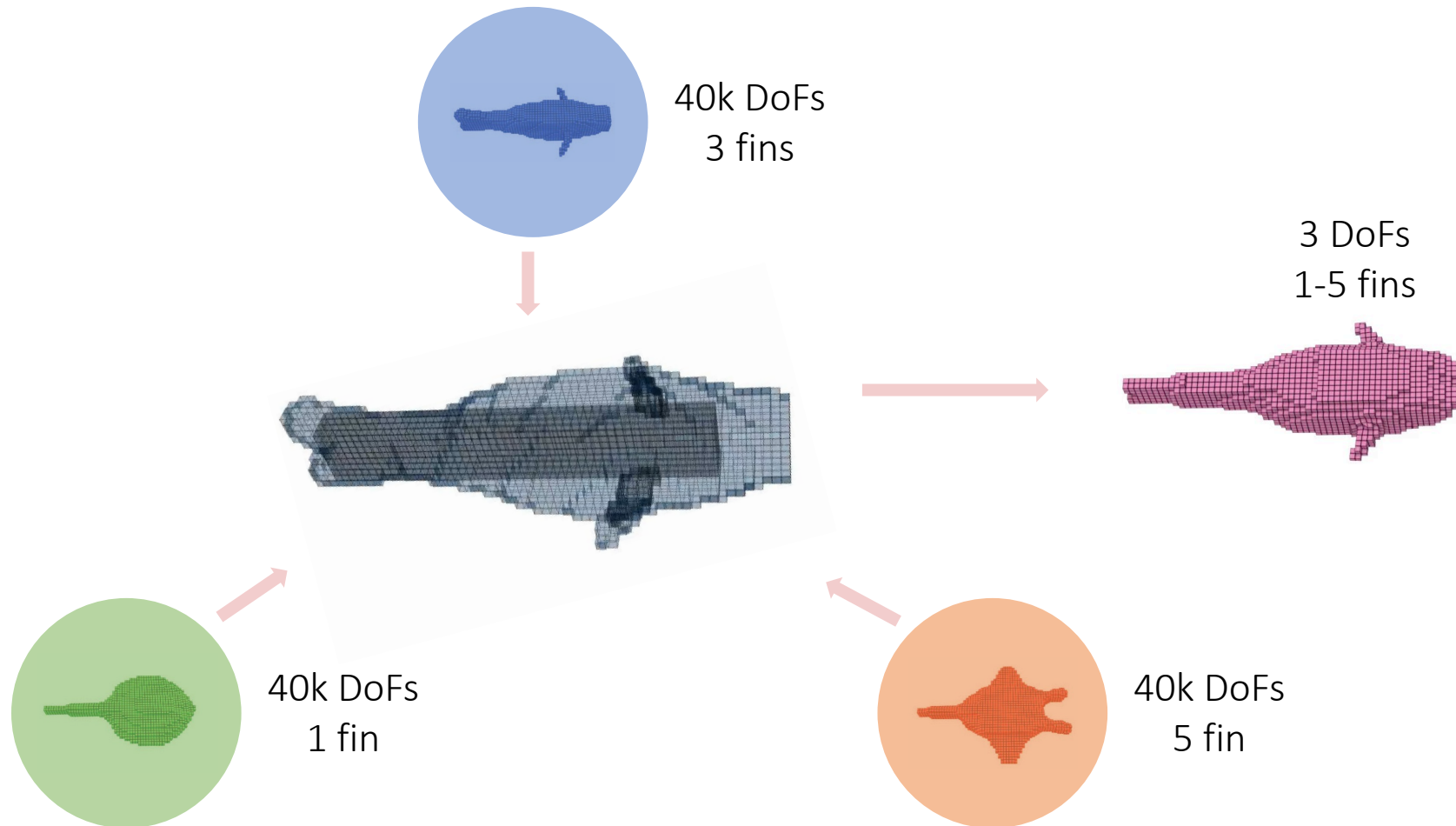


40k DoFs  
5 fin

# Our approach: Wasserstein gradients

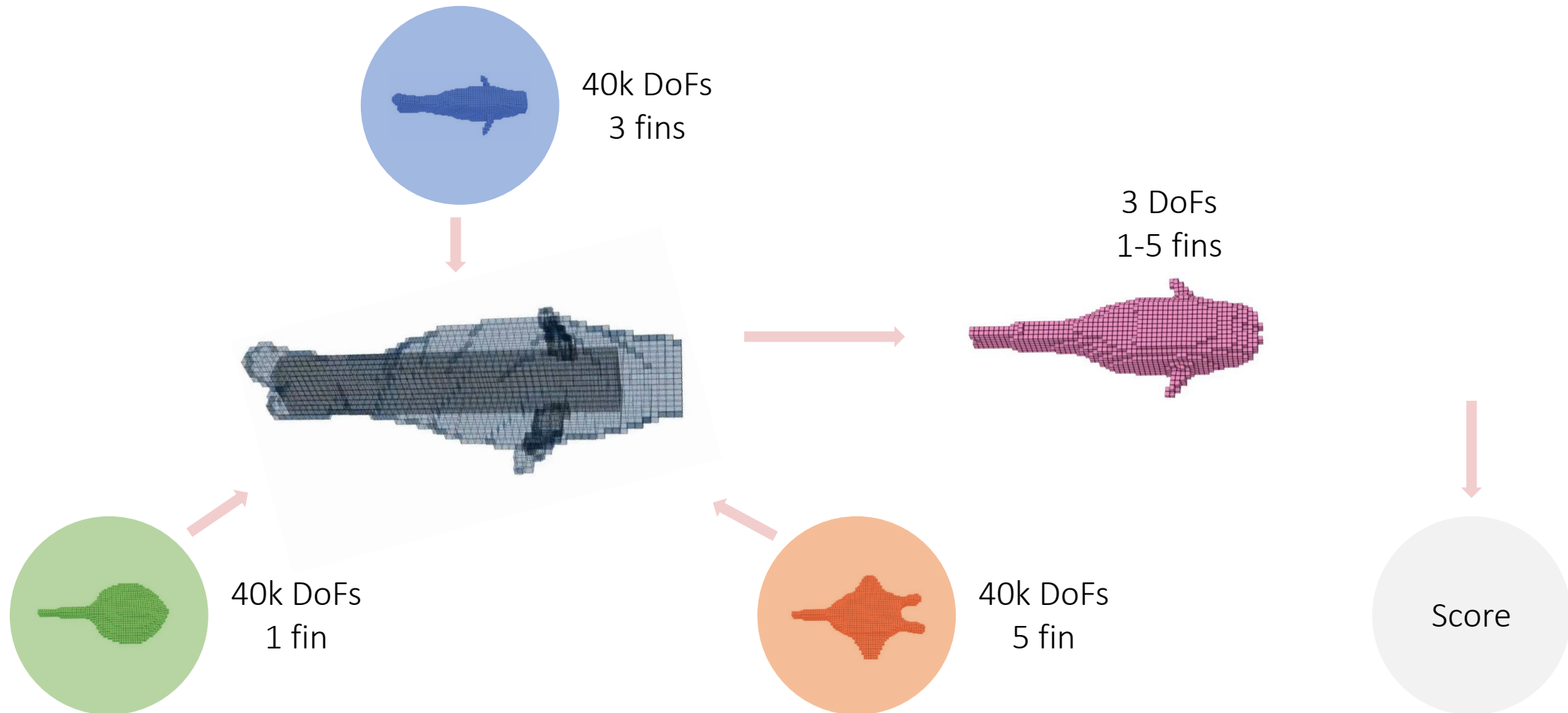


# Our approach: Wasserstein gradients

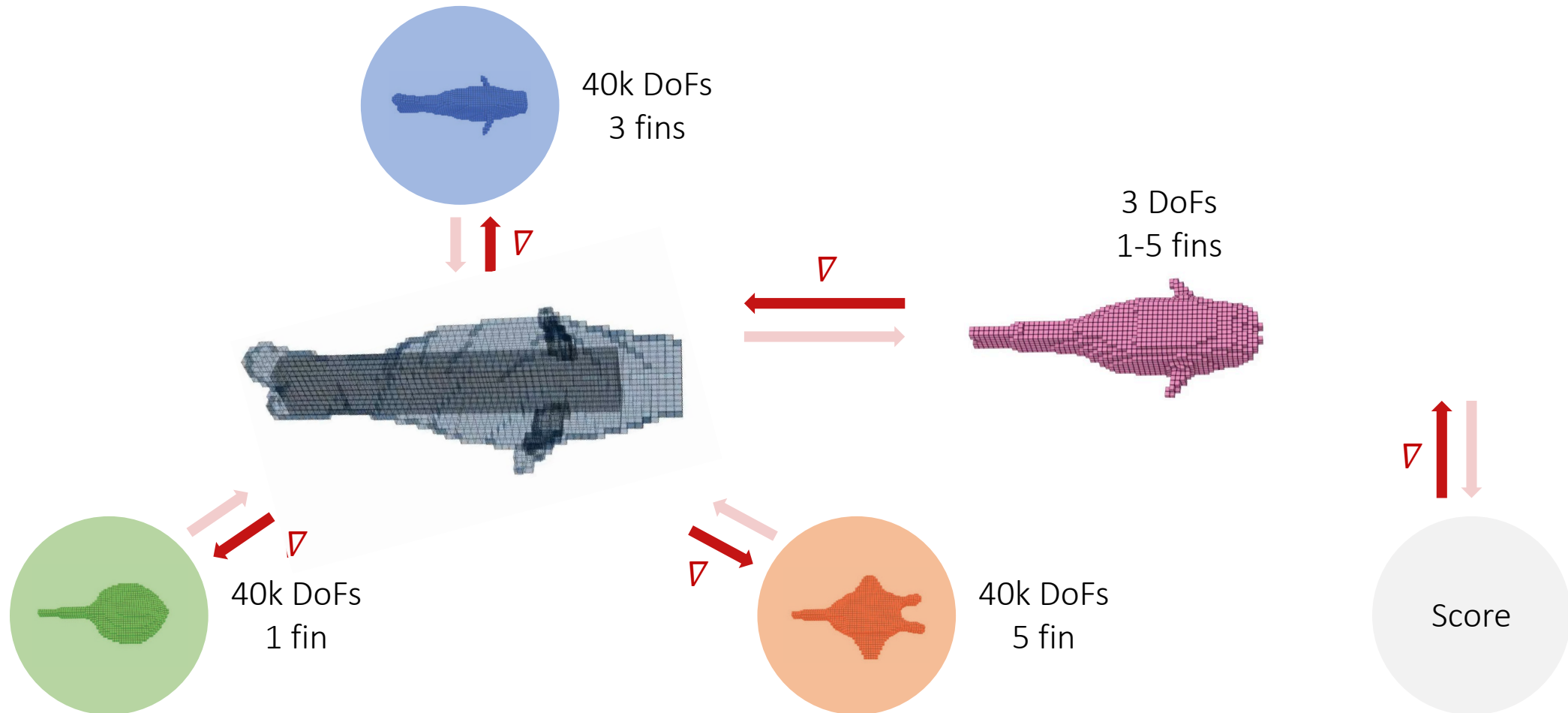




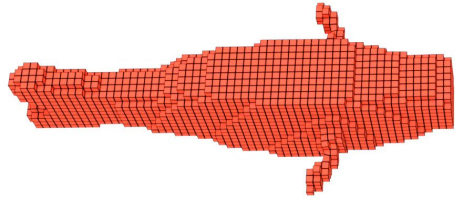
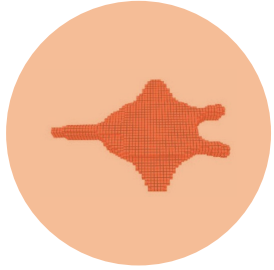
# Our approach: Wasserstein gradients



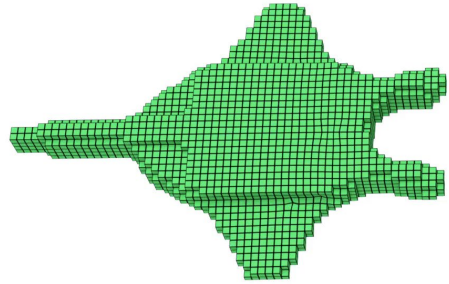
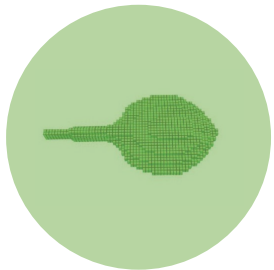
# Our approach: Wasserstein gradients



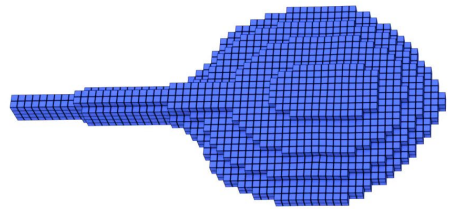
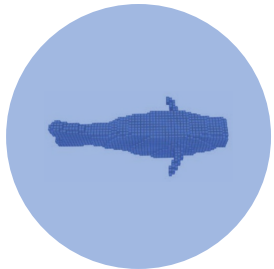
# Example: speedy fish



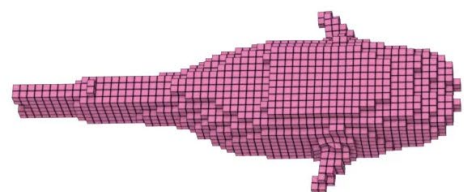
Optimized control only



Optimized control only

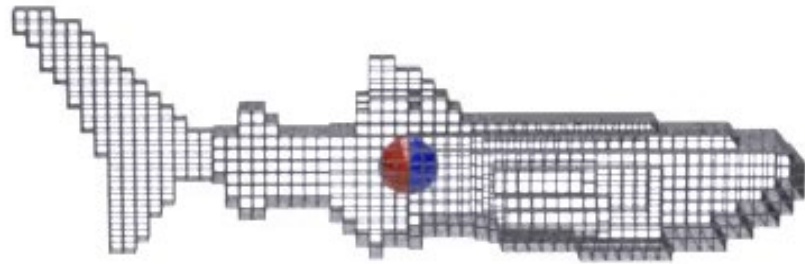


Optimized control only

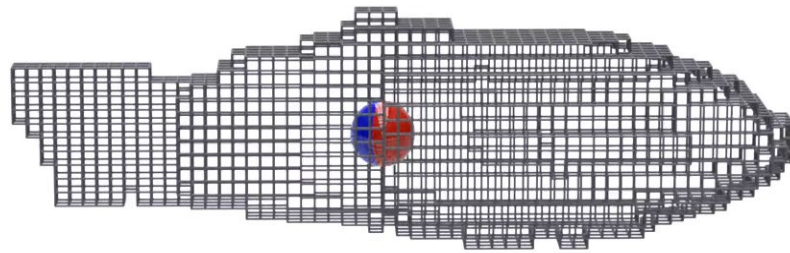


Optimized shape and control

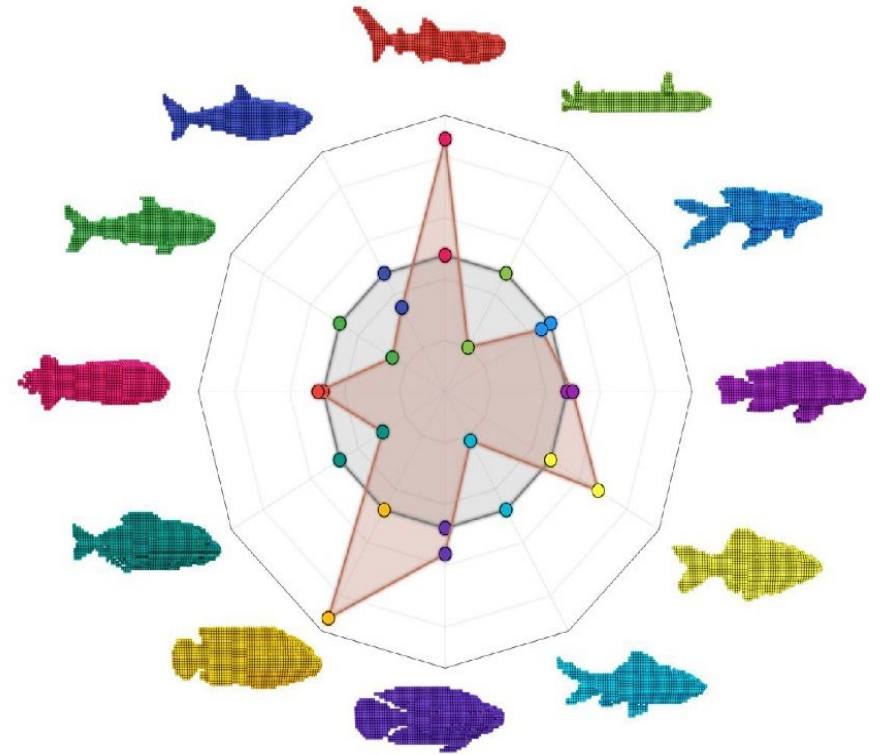
# Example: flow-resistant fish



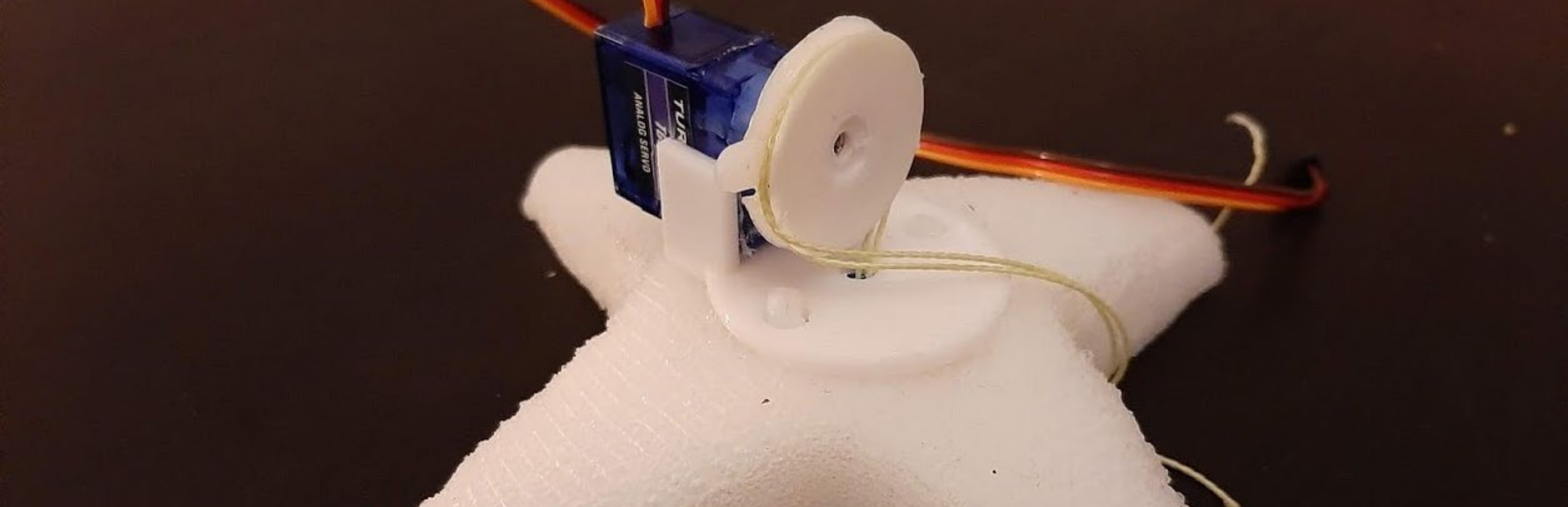
Unoptimized



Optimized



Wasserstein weights



# Underwater Soft Robot Modeling and Control with Differentiable Simulation

Tao Du\*, Josie Hughes\*, Sebastien Wah, Wojciech Matusik, Daniela Rus

IEEE RA-L/RoboSoft 2021

# Summary

## ▽ Parametrization

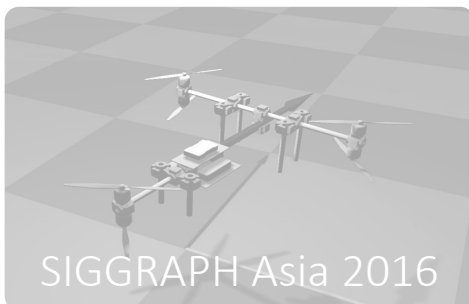
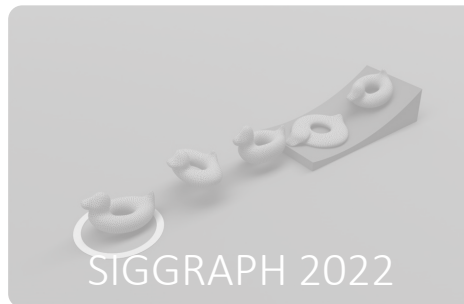
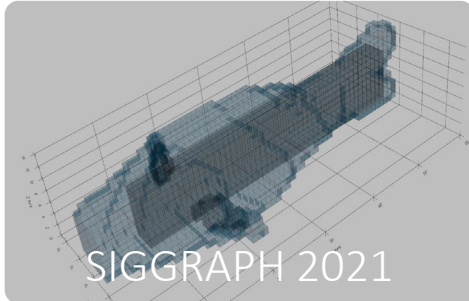
Initializing parameters

## ▽ Modeling

Deriving governing equations

## ▽ Evaluation

Computing performance metrics





# Summary

## ▽ Parametrization

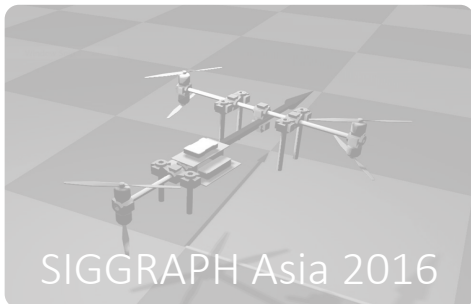
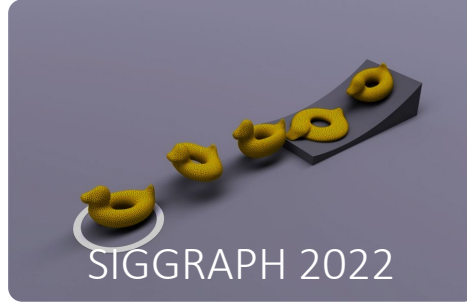
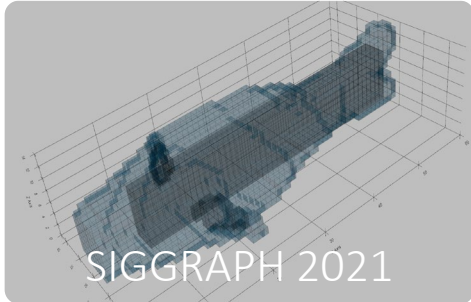
Initializing parameters

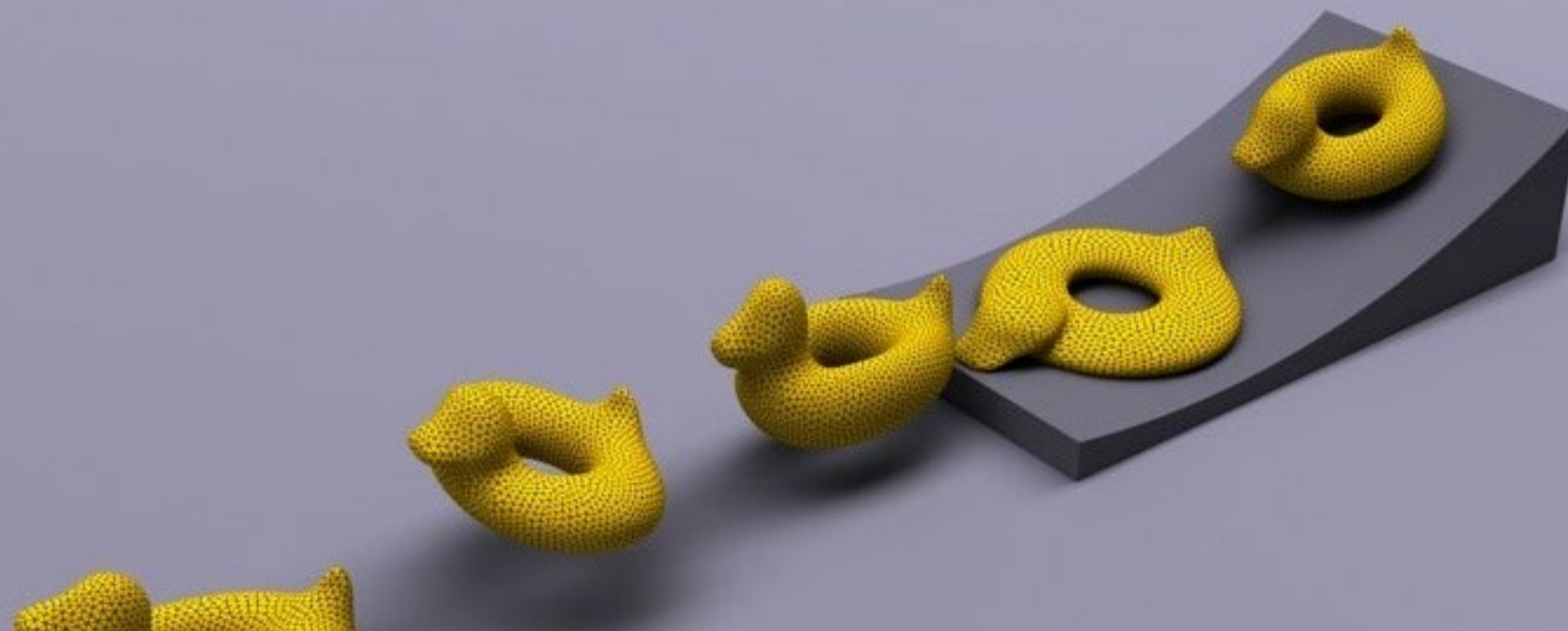
## ▽ Modeling

Deriving governing equations

## ▽ Evaluation

Computing performance metrics





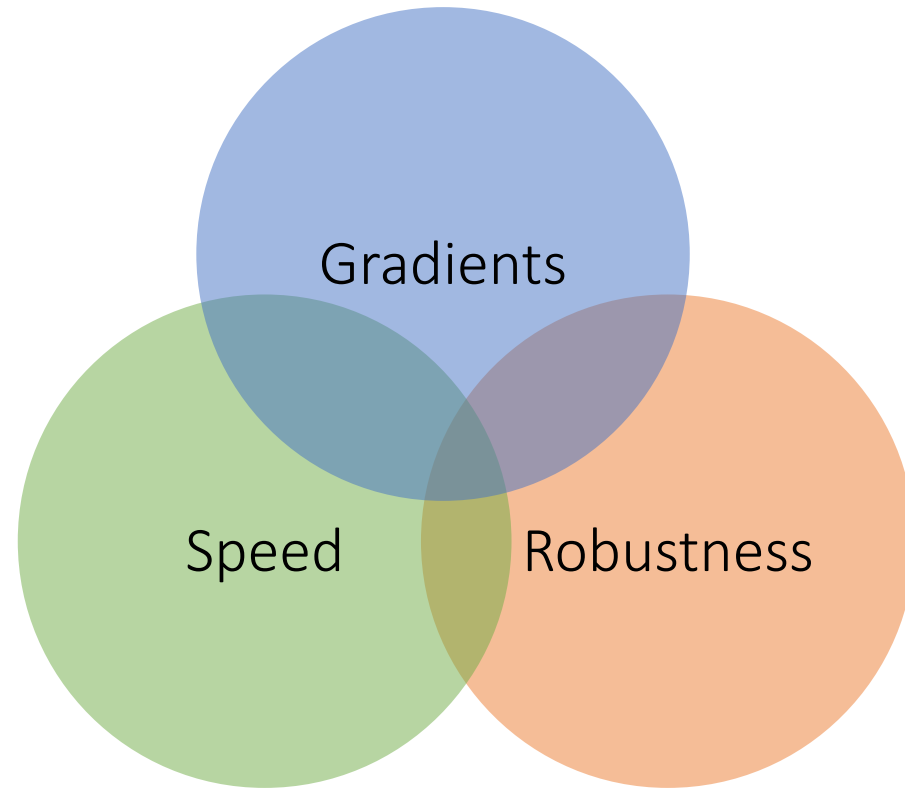
# DiffPD: Differentiable Projective Dynamics

Tao Du, Kui Wu, Pingchuan Ma, Sebastien Wah, Andrew Spielberg, Daniela Rus, Wojciech Matusik

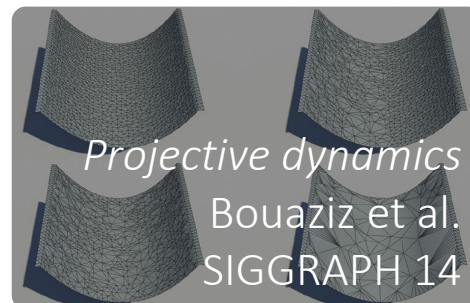
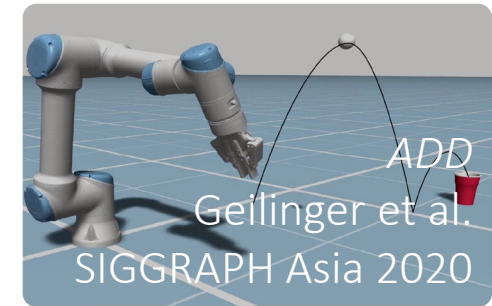
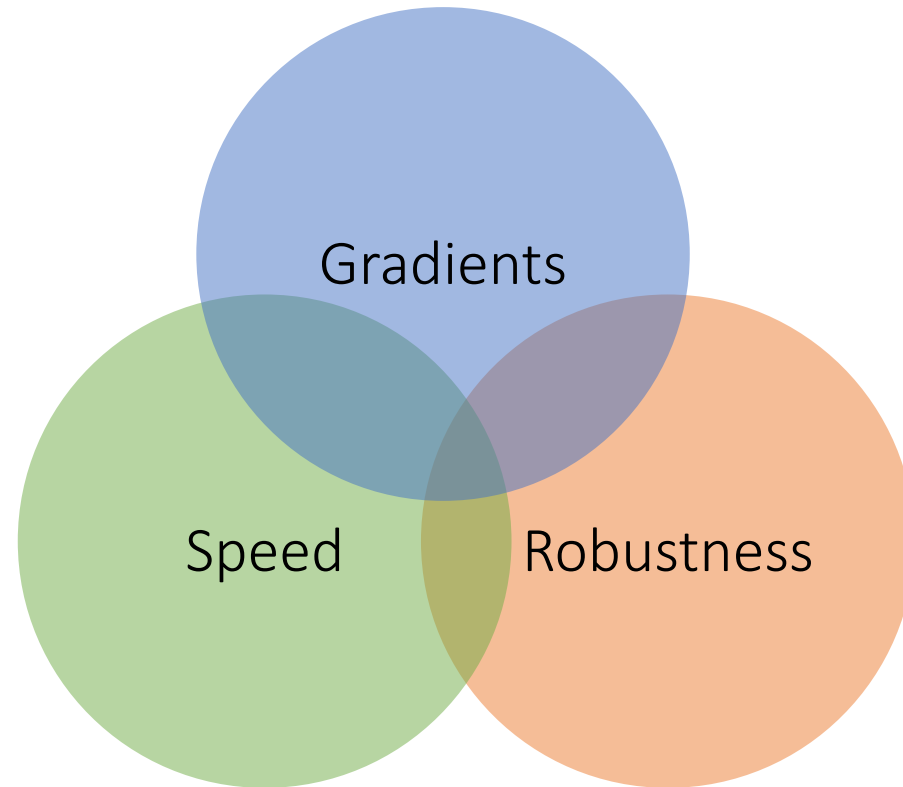
ACM Transactions on Graphics (SIGGRAPH 2022)



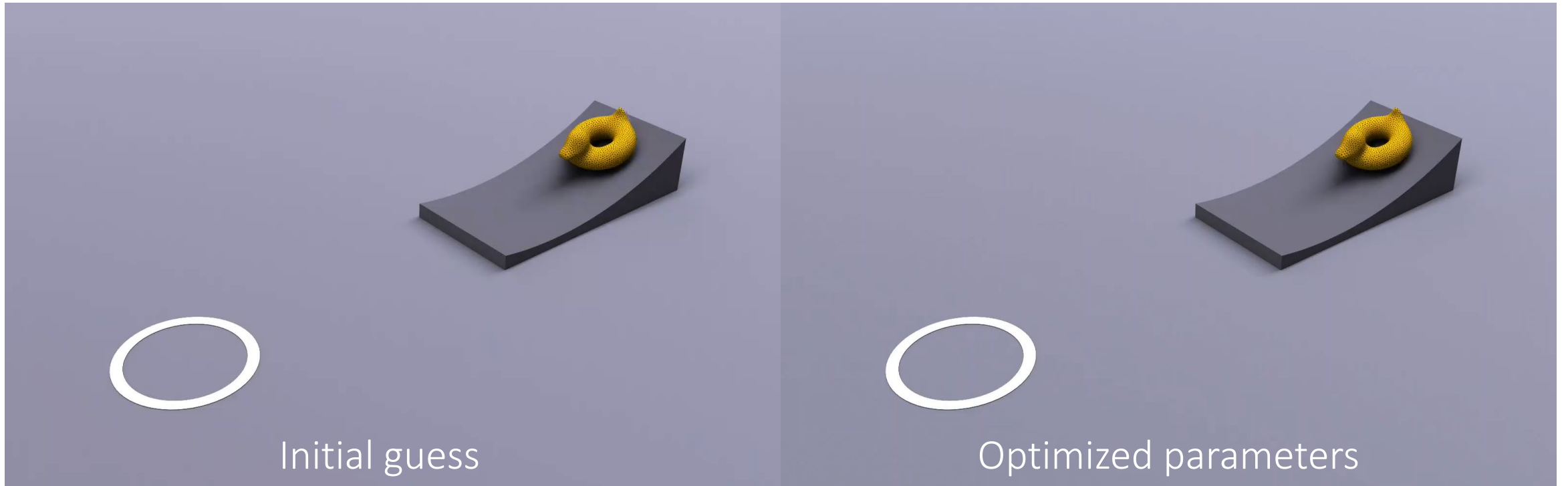
# Feature highlights



# Feature highlights



# Applications: system identification

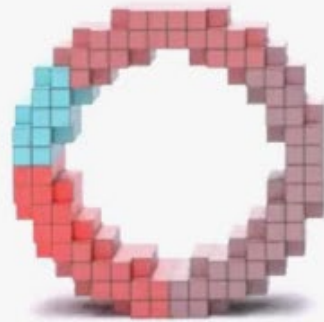


# Applications: trajectory optimization

Initial guess

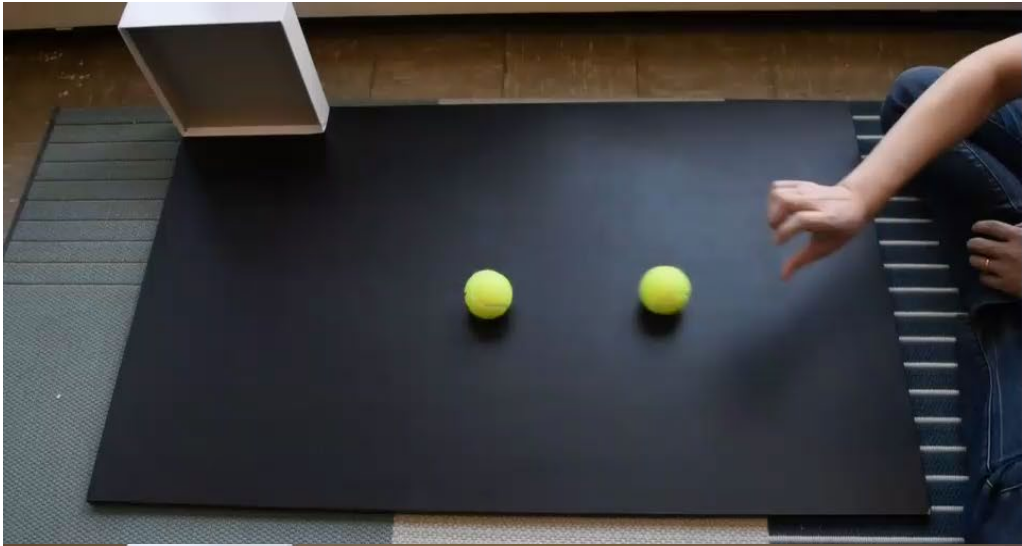


Optimized parameters

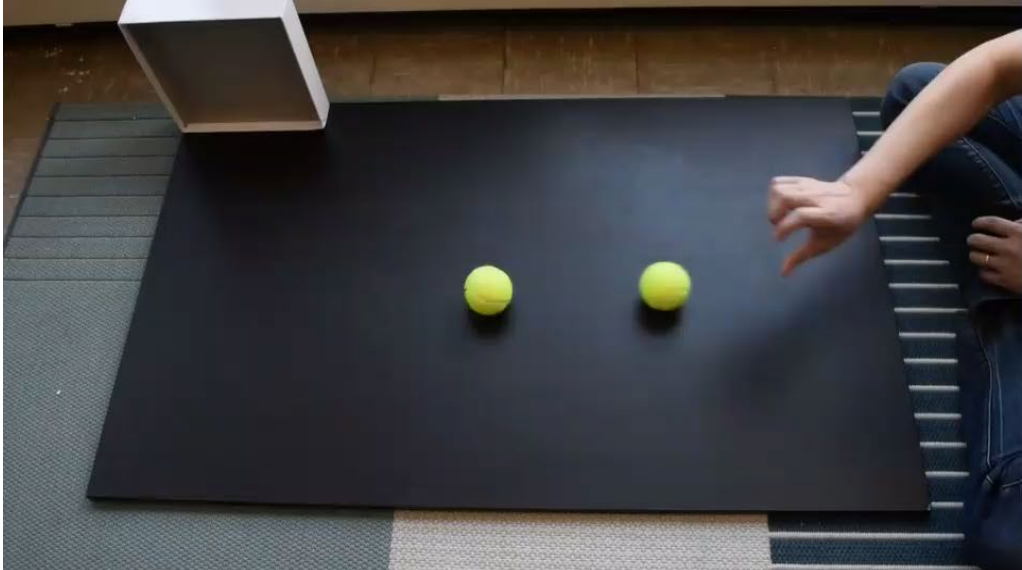
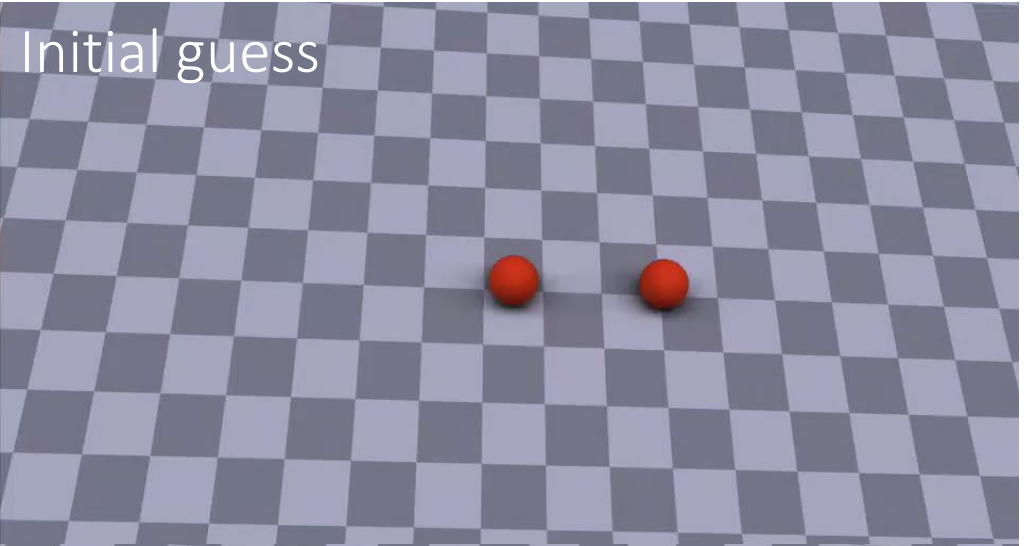


Contraction  Expansion

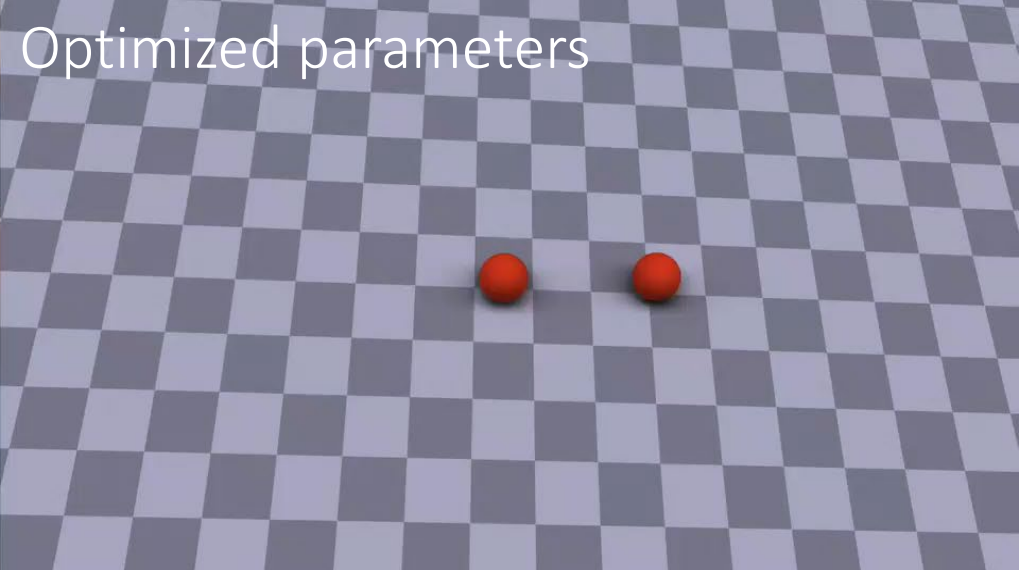
# Applications: real-to-sim transfer



Initial guess



Optimized parameters



# Summary

## ▽ Parametrization

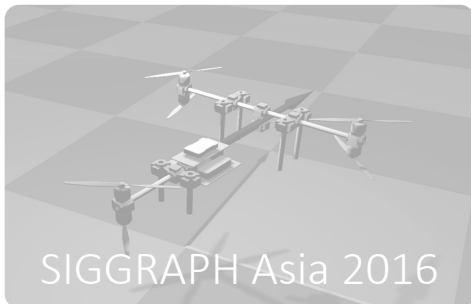
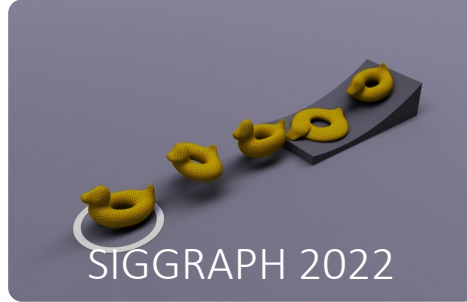
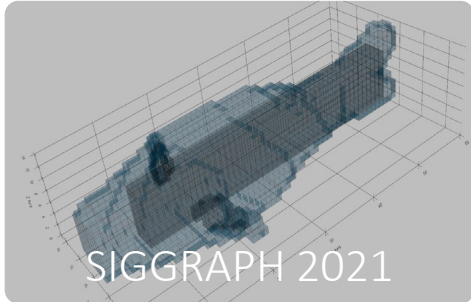
Initializing parameters

## ▽ Modeling

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# Summary

## ▽ Parametrization

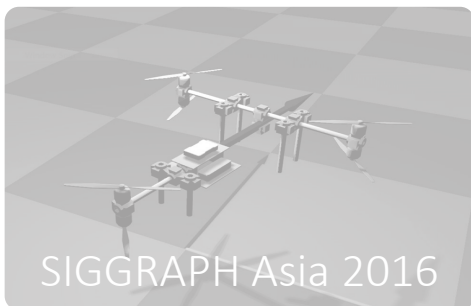
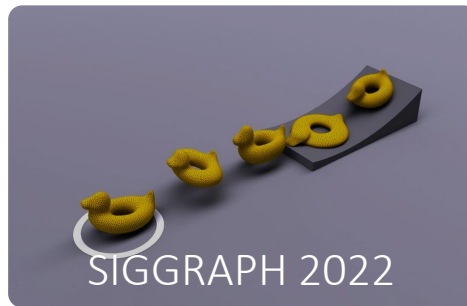
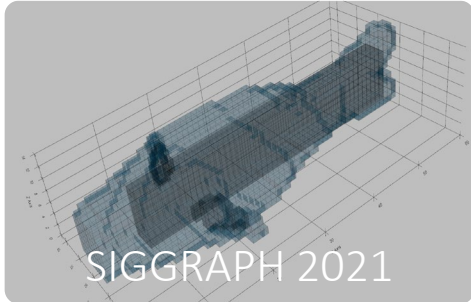
Initializing parameters

## ▽ Modeling

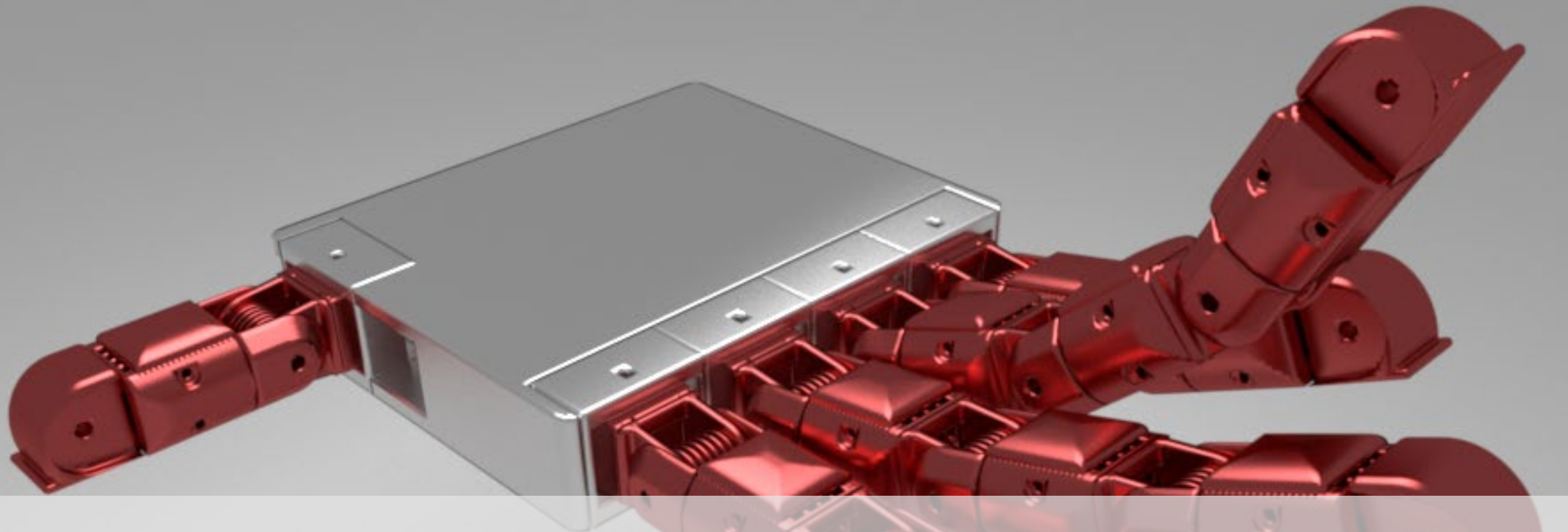
Deriving governing equations

## ▽ Evaluation

Computing performance metrics







# **RISP: Rendering-Invariant State Predictor with Differentiable Simulation and Rendering for Cross-Domain Parameter Estimation**

Pingchuan Ma\*, Tao Du\*, Joshua B. Tenenbaum, Wojciech Matusik, Chuang Gan

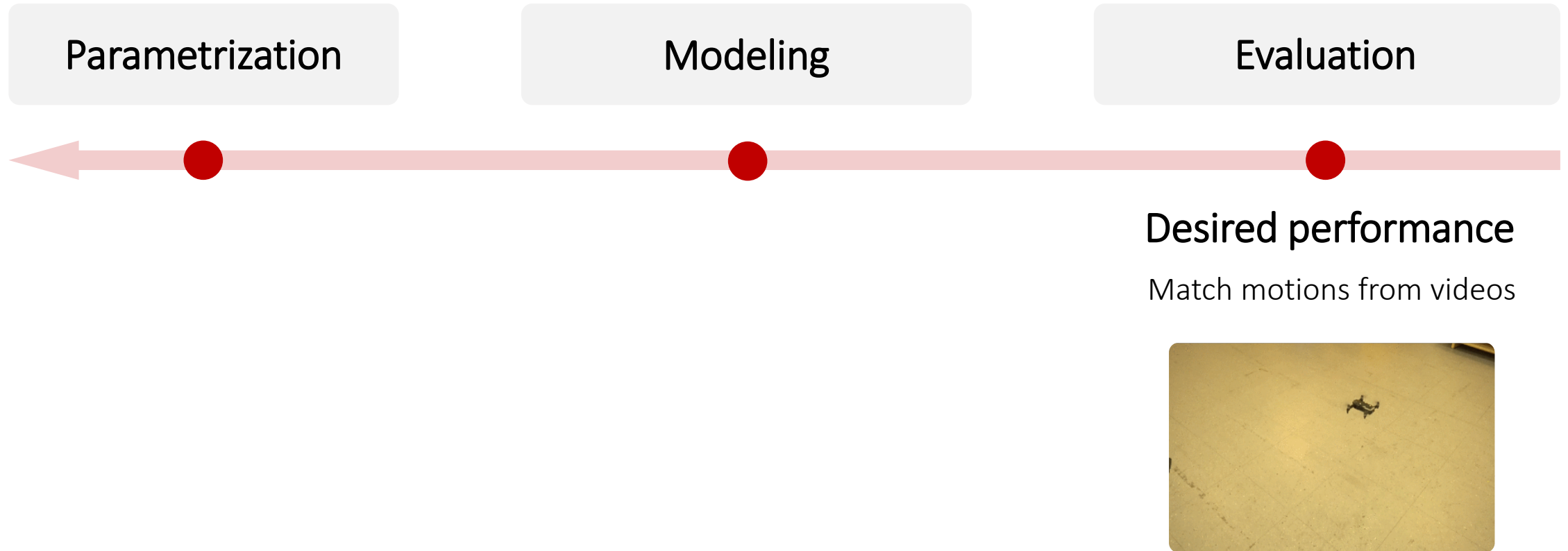
ICLR 2022 (oral)



# Problem statement

Build a digital twin of a robot from its video of motion sequences.

# Why is it an inverse dynamics problem



# Why is it an inverse dynamics problem

Parametrization

Modeling

Evaluation



Known dynamic model

Euler-Lagrange dynamics

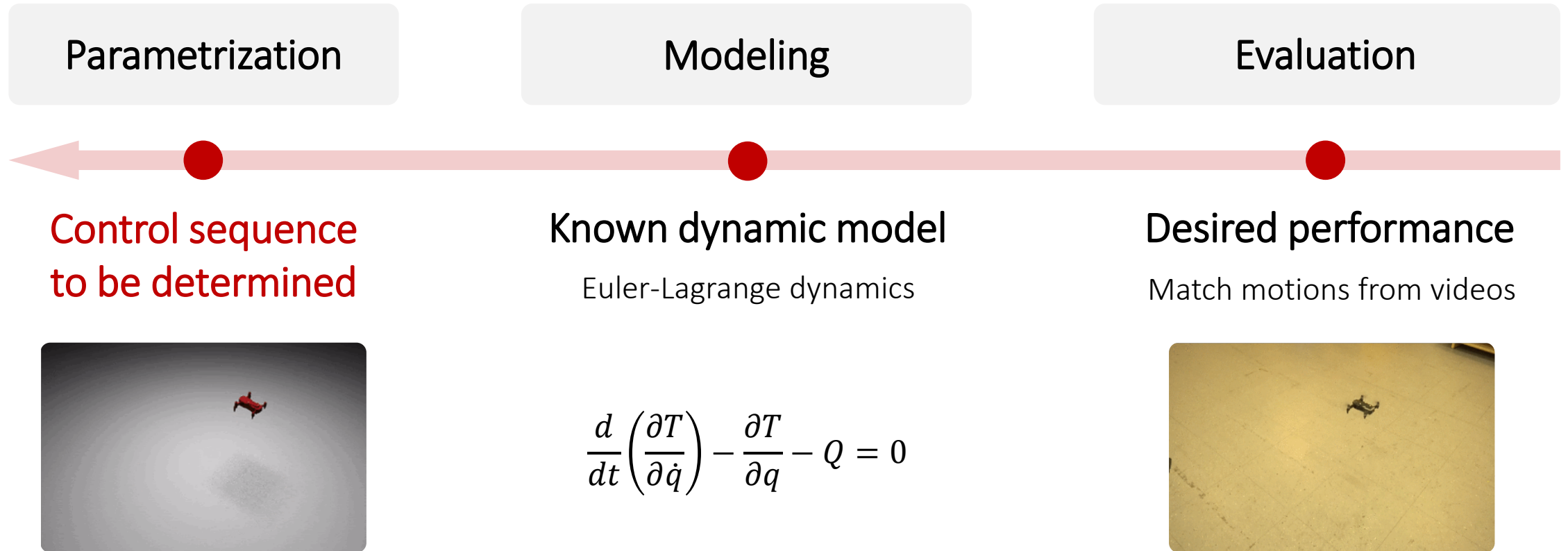
$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} - Q = 0$$

Desired performance

Match motions from videos

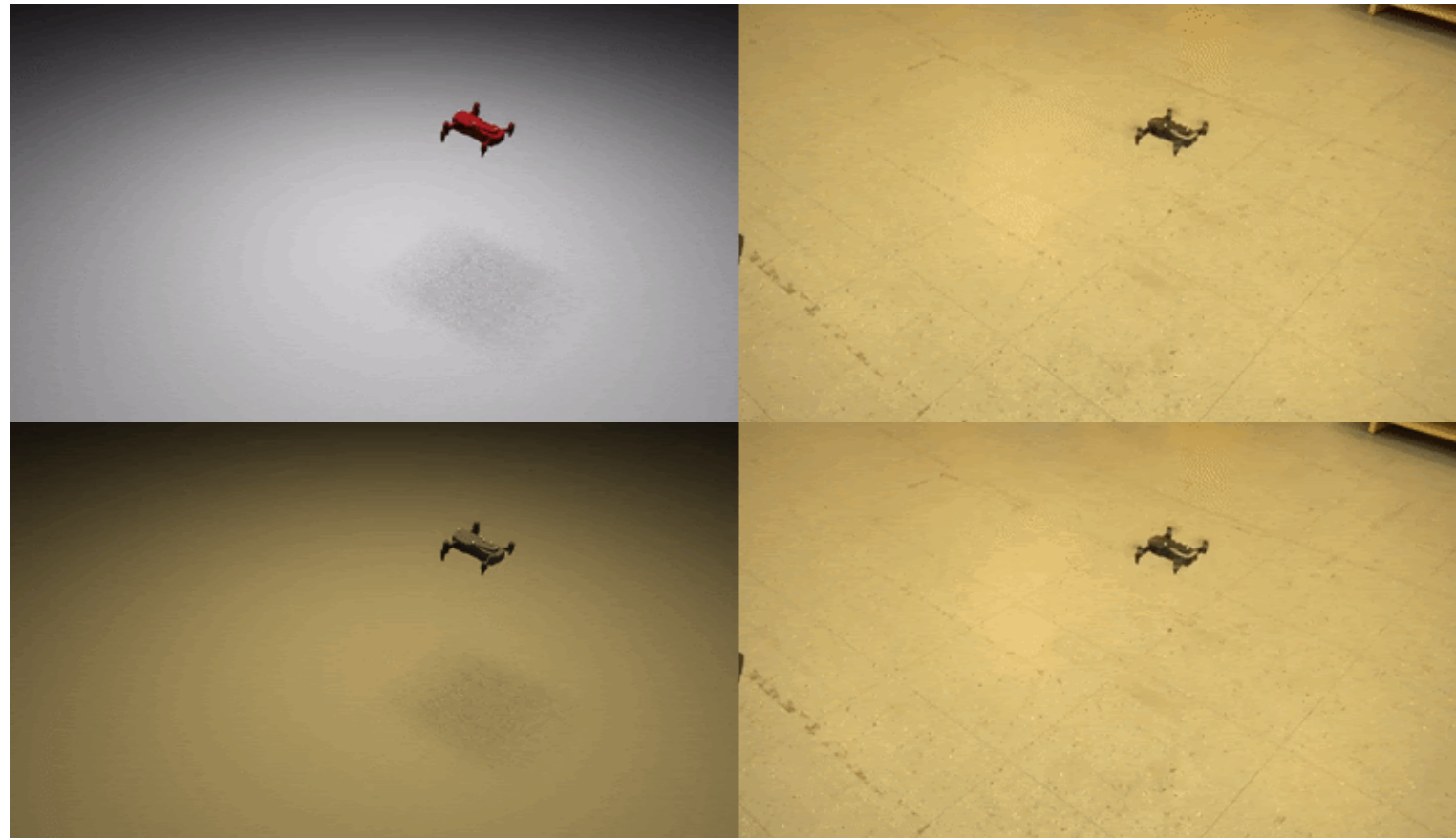


# Why is it an inverse dynamics problem

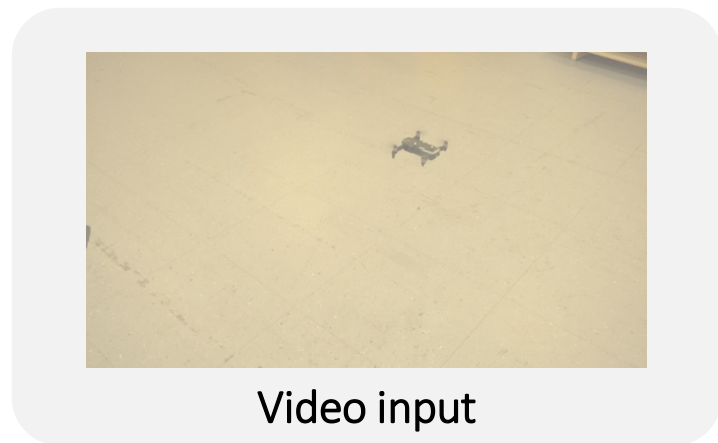


# The challenge

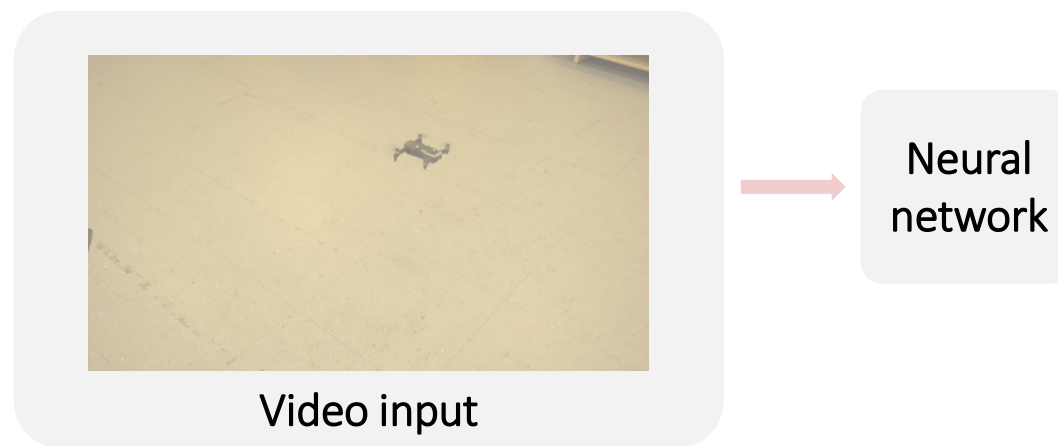
The unknown visual appearance parameters shadows the dynamics information.



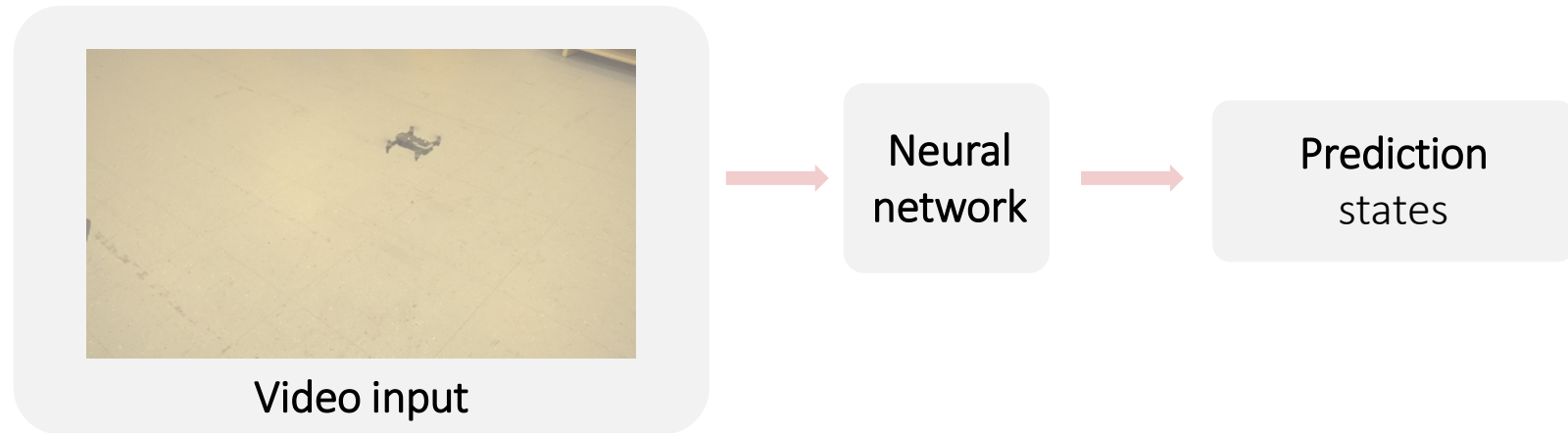
# The first idea: a state-prediction network



# The first idea: a state-prediction network

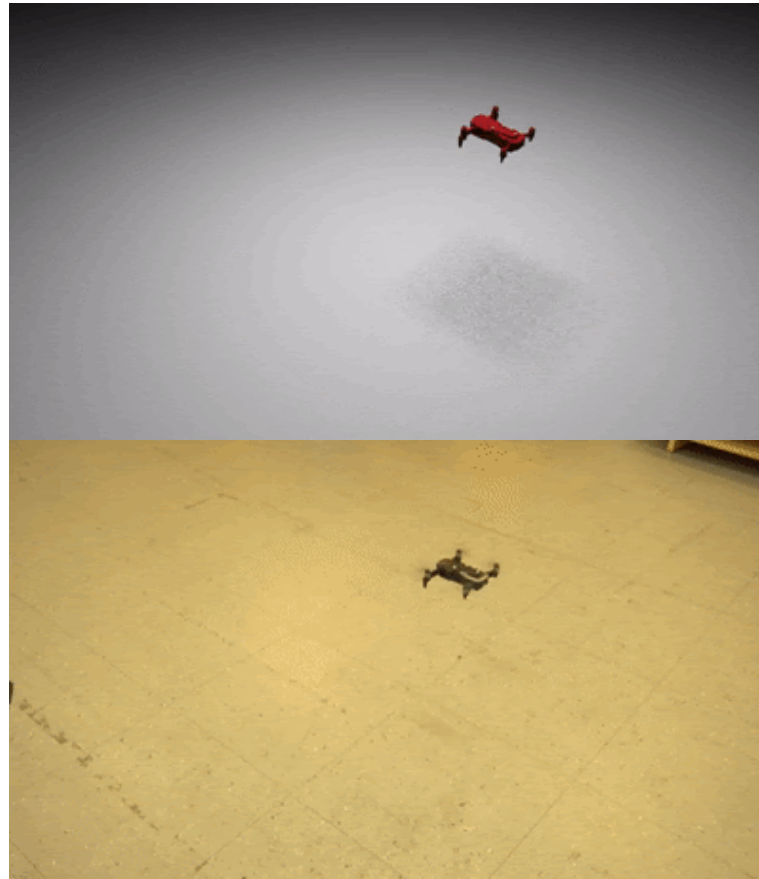


# The first idea: a state-prediction network

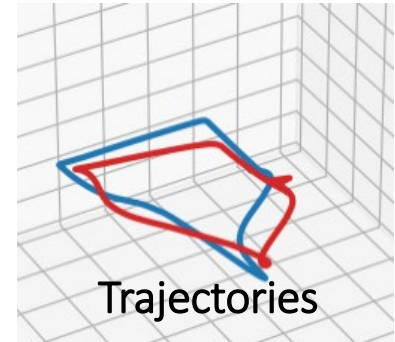




# The first idea: a state-prediction network

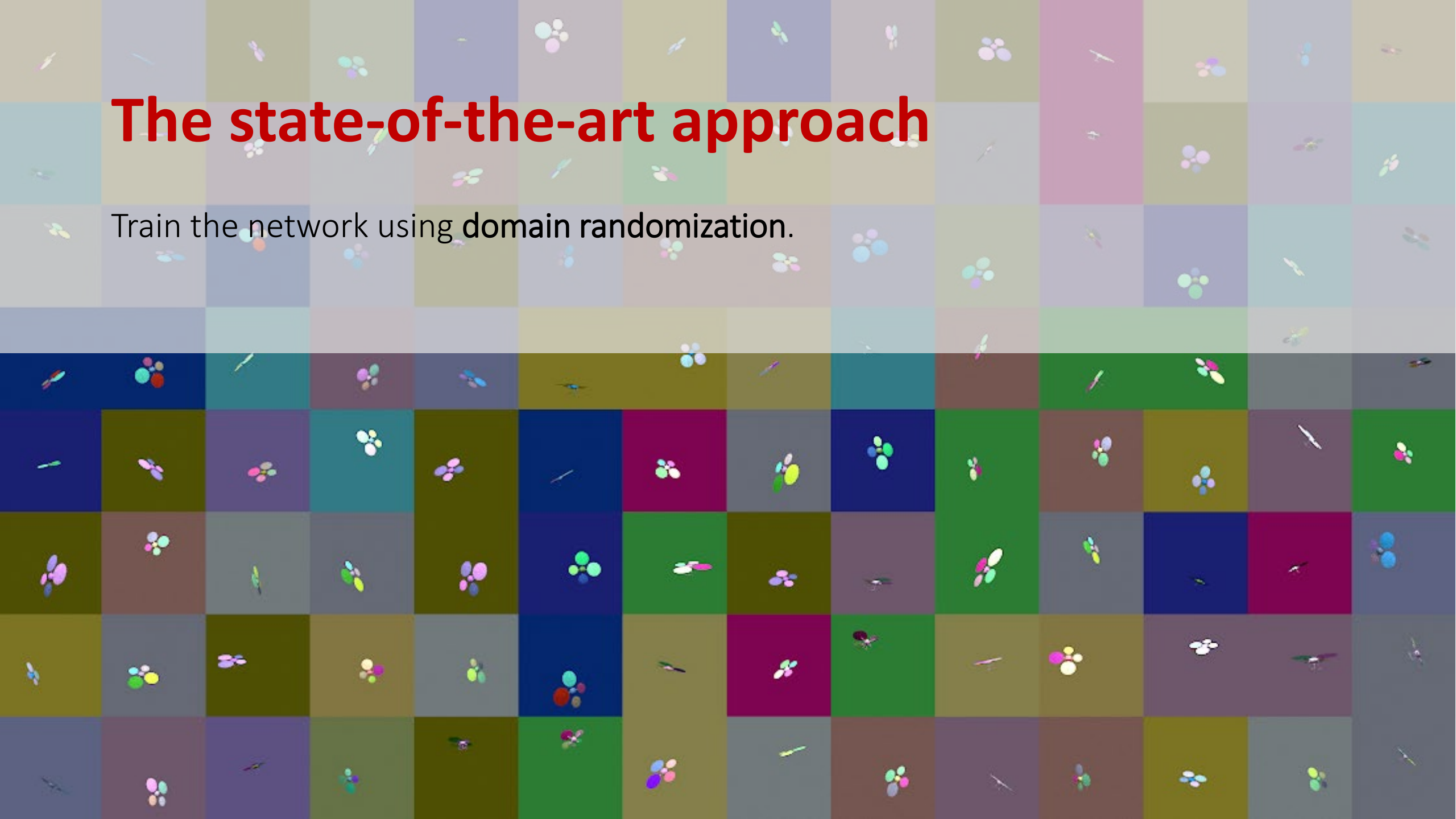


Neural  
network

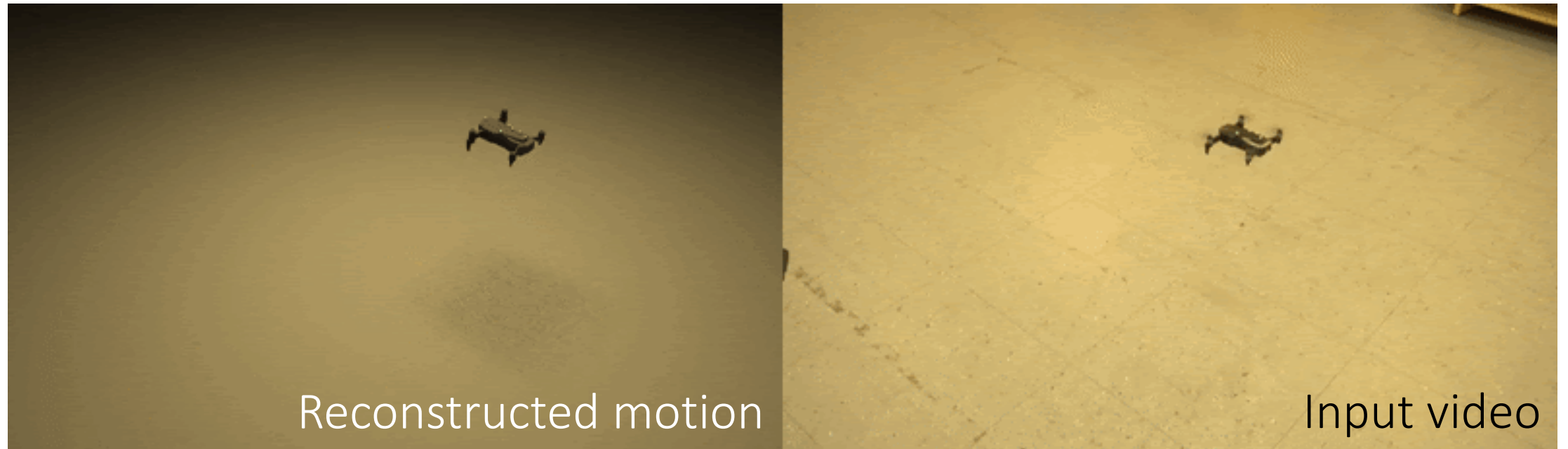


# The state-of-the-art approach

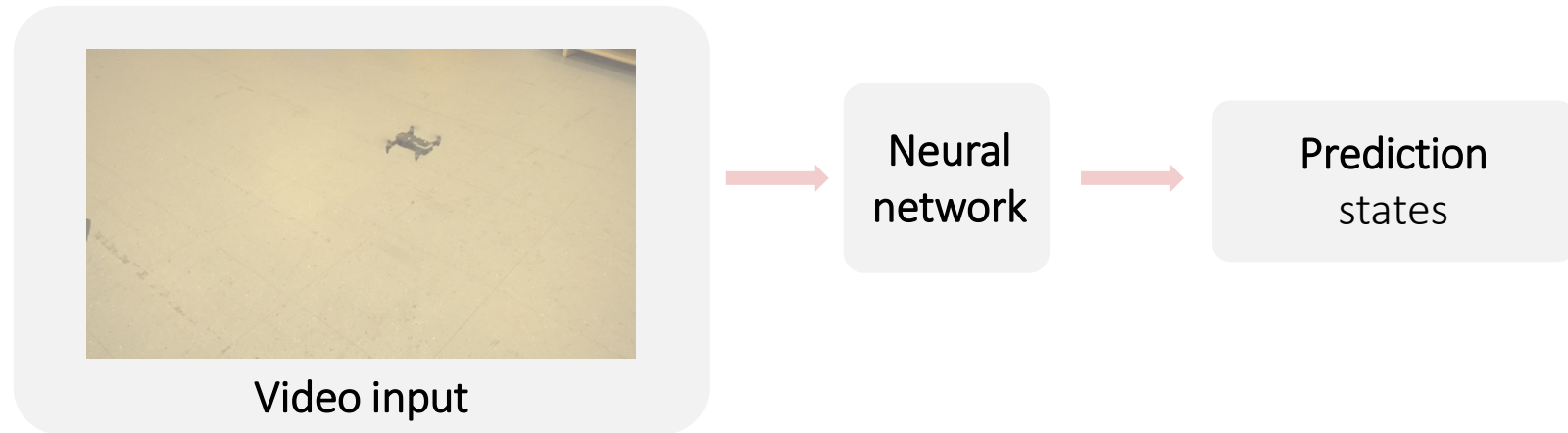
Train the network using domain randomization.



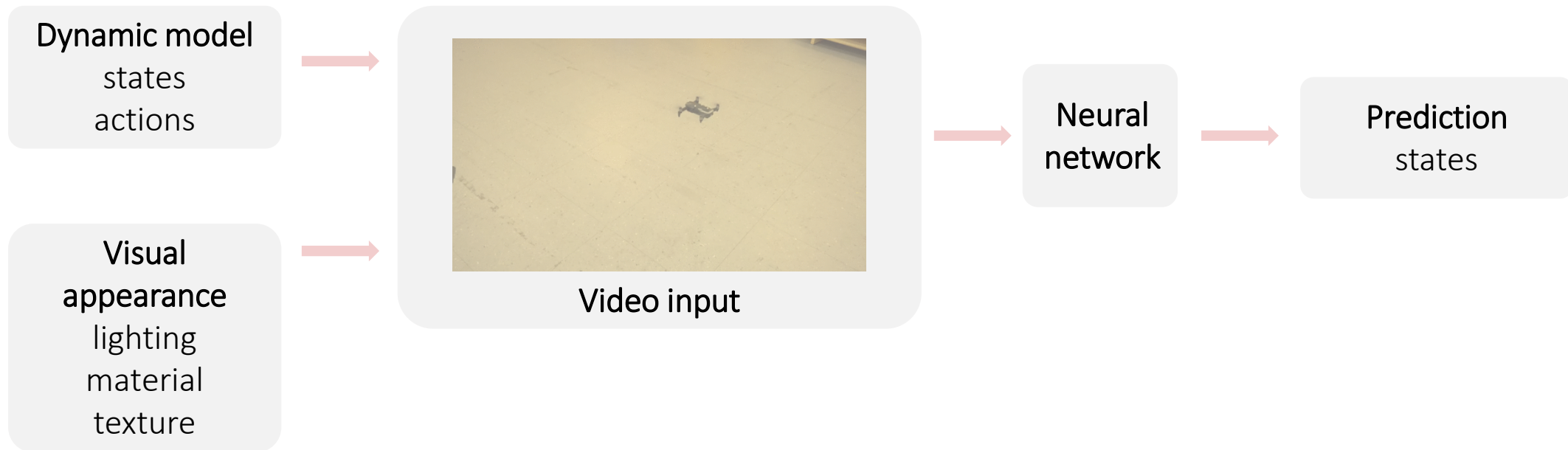
# Domain randomization failed here...



# Why did domain randomization fail?

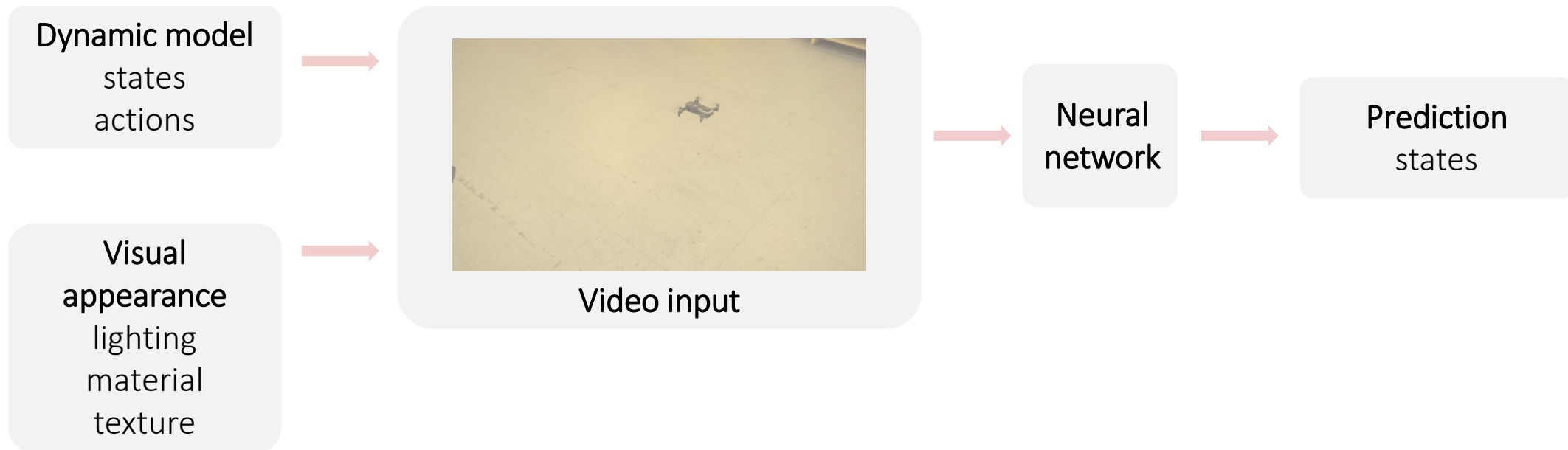


# Why did domain randomization fail?



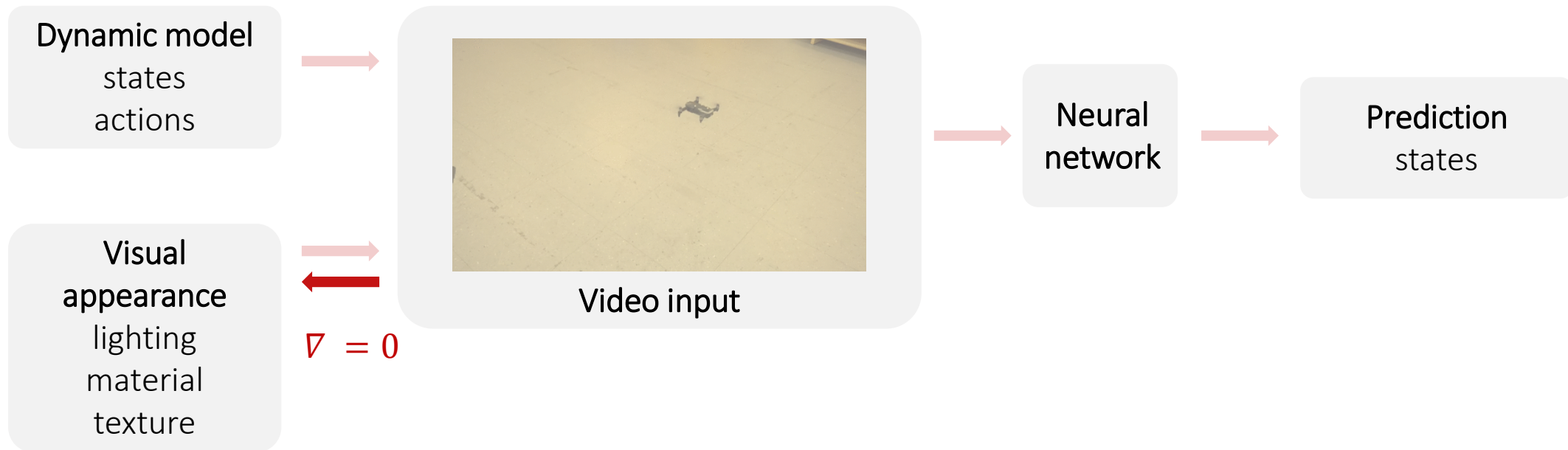
# Why did domain randomization fail?

The network needs to maintain invariance under different visual appearances.



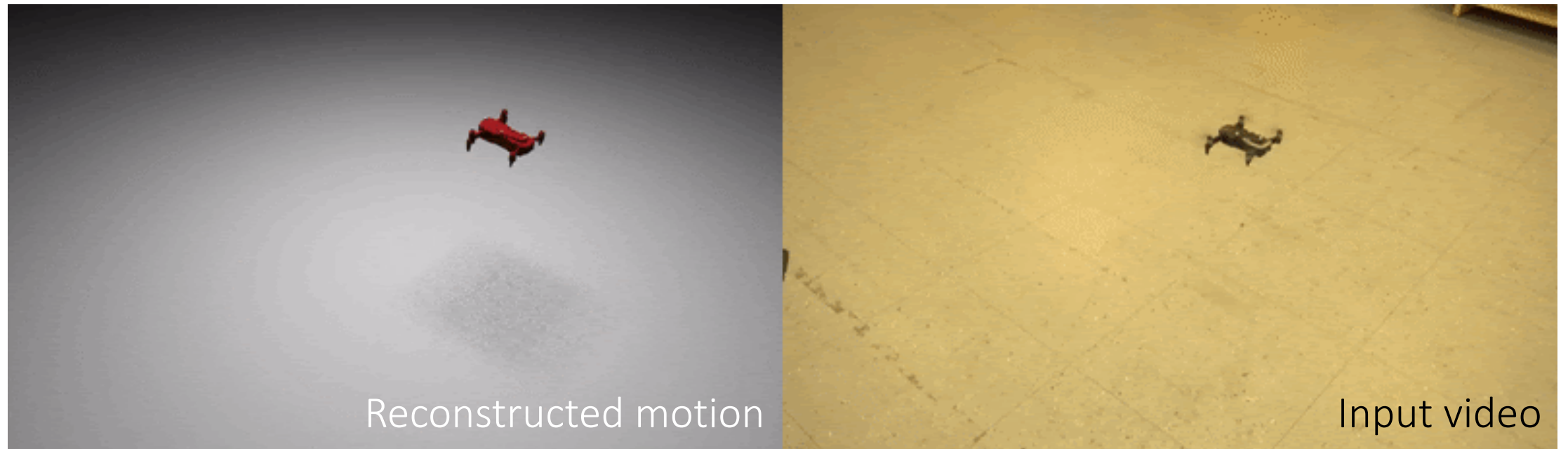
# The second idea: rendering-invariance

The network needs to maintain invariance under different visual appearances.



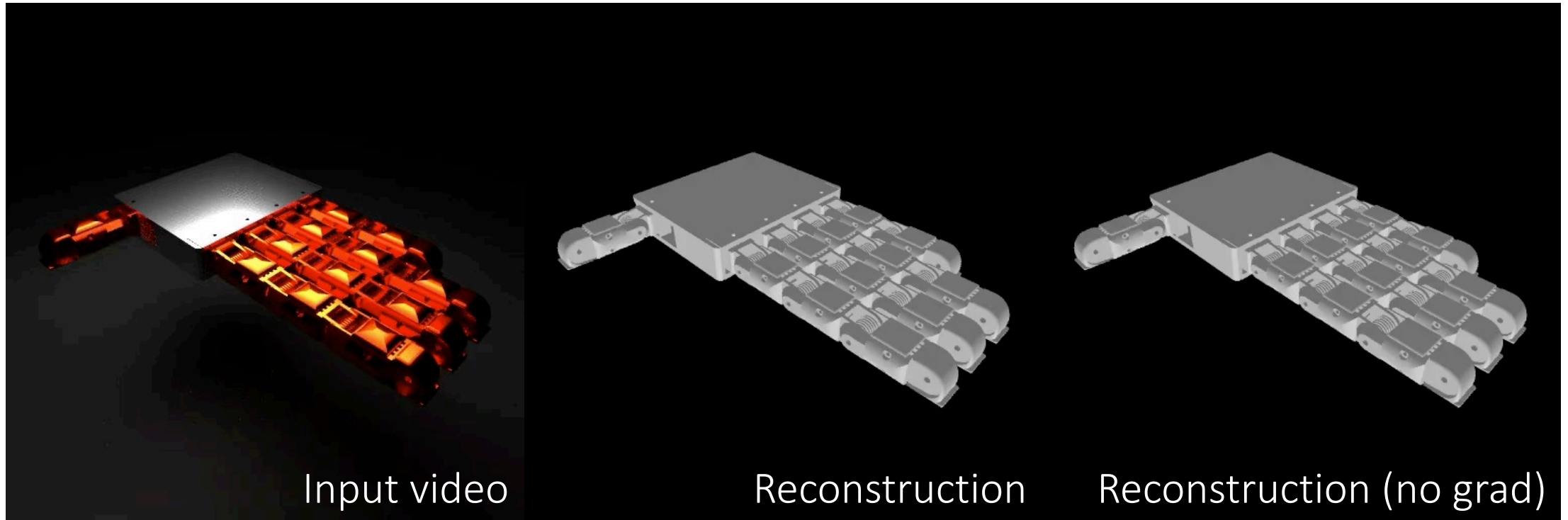


# Results: quadrotors



Note that the rendering configuration is intentionally made different.

# Results: dexterous hand



Note that the rendering configuration is intentionally made different.

# Summary

## ▽ Parametrization

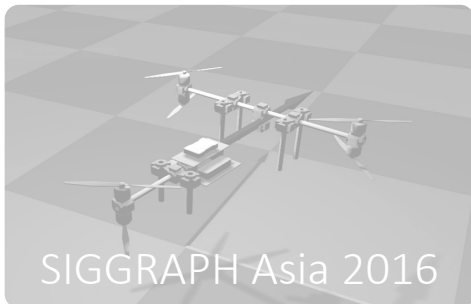
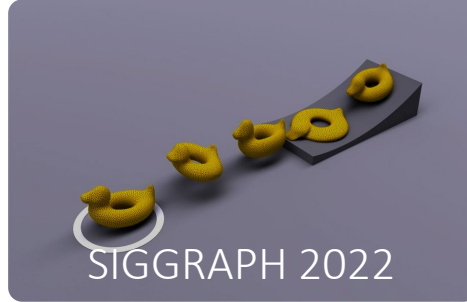
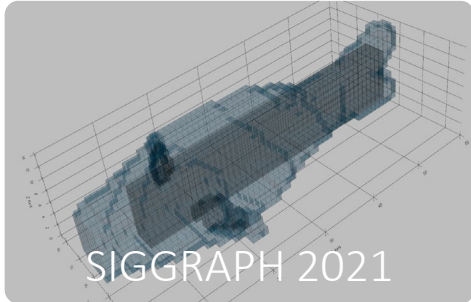
Initializing parameters

## ▽ Modeling

Deriving governing equations

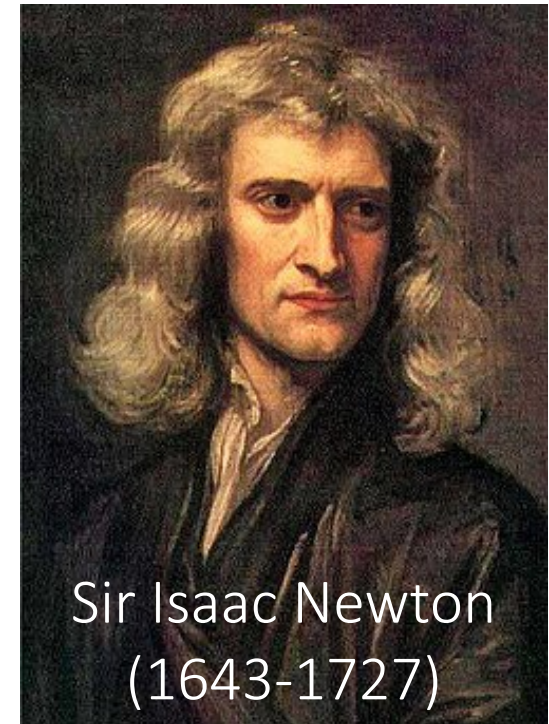
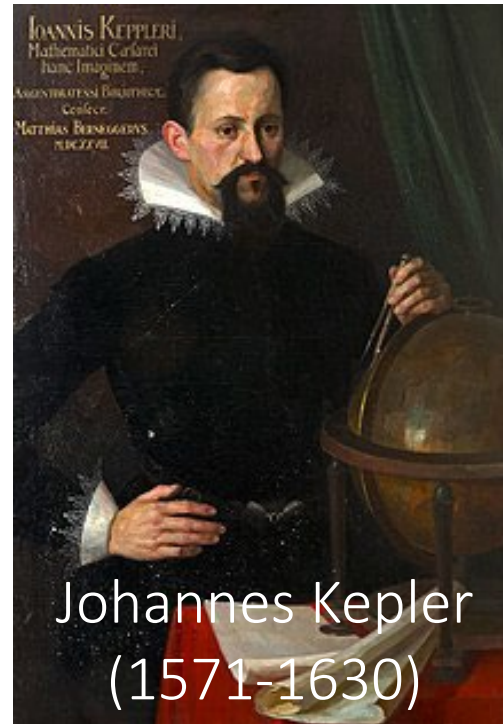
## ▽ Evaluation

Computing performance metrics



# What is next?

Let me end the talk with what I consider one of the most inspiring inverse dynamics problems in history.



# What is next?

The most rewarding inverse problem is to discover scientific laws.

# Acknowledgment

The papers covered in this talk were funded by the following sponsors:





# Acknowledgment

Page 2: ANYmal: <https://rsl.ethz.ch/robots-media/anymal.html>.

Page 2: SCLS: [http://www.scls.riken.jp/en/research/01\\_dynamics/index.html](http://www.scls.riken.jp/en/research/01_dynamics/index.html).

Page 2: Ravuri, S., Lenc, K., Willson, M. et al. *Skilful precipitation nowcasting using deep generative models of radar*. Nature 597, 672–677 (2021). <https://doi.org/10.1038/s41586-021-03854-z>.

Page 3: Natural Portfolio. <https://www.nature.com/subjects/dynamical-systems>.

Page 14: Pfaff et al. *Learning mesh-based simulation with graph networks*. ICLR 2021.

Page 14: Chen et al. *A system for general in-hand object re-orientation*. CoRL 2021.

Page 17: Hahn et al. *Real2Sim: visco-elastic parameter estimation from dynamic motion*. SIGGRAPH Asia 2019.

Page 17: Peng et al. *DeepMimic: Example-guided deep reinforcement learning of physics-based character skills*. SIGGRAPH 2018.

Page 25: Katzschmann et al. *Exploration of underwater life with an acoustically controlled soft robotic fish*. Science Robotics 2018.

Page 25: Project CETI. <https://www.projectceti.org/>.



# Acknowledgment

Page 29: Video credit to Jie Xu.

Page 42: Hu et al. *ChainQueen: A real-time differentiable physical simulator for soft robotics*. ICRA 2019.

Page 42: Bouaziz et al. *Projective dynamics: Fusing constraint projections for fast simulation*. SIGGRPAH 2014.

Page 42: Geilinger et al. *ADD: Analytically differentiable dynamics for multi-body systems with frictional contact*. SIGGRAPH Asia 2020.

Page 67: Tycho Brahe. [https://en.wikipedia.org/wiki/Tycho\\_Brahe](https://en.wikipedia.org/wiki/Tycho_Brahe).

Page 67: Johannes Kepler: [https://en.wikipedia.org/wiki/Johannes\\_Kepler](https://en.wikipedia.org/wiki/Johannes_Kepler).

Page 67: Isaac Newton: [https://en.wikipedia.org/wiki/Isaac\\_Newton](https://en.wikipedia.org/wiki/Isaac_Newton).

# Thank you!

Papers, code, and data are available at <https://people.csail.mit.edu/taodu>

Email: [taodu@csail.mit.edu](mailto:taodu@csail.mit.edu)