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Exploring Business Models and Dynamic Pricing Frameworks for SPOC Services

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MOOC platforms

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- Coursera: 3133 courses
- EdX: 2293 courses
- XuetangX: 1507 courses

How do they generate revenue?

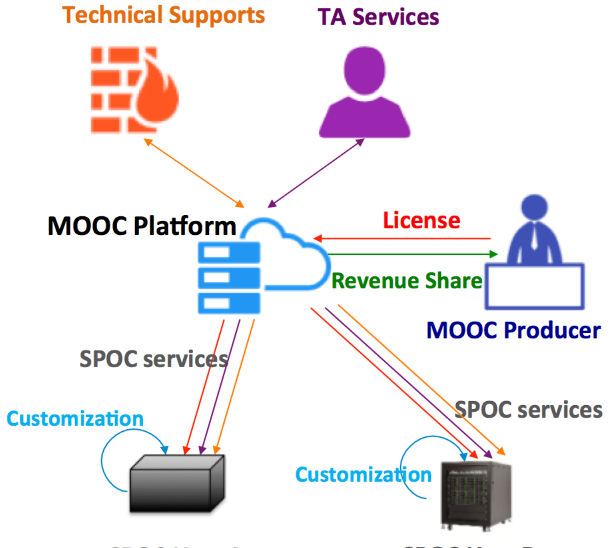
B2C (Business-to-Customer)

- Verified Certificates
- Specializations
- Online Micro Masters
- Advanced Placement

B2B (Business-to-Business)

- sub-licensing MOOC contents
- on-campus SPOC platforms

SPOC services



Why do we need an auction?

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A Bundle of User's Demand

- MOOC contents
- Teaching assistant services
- SaaS services
- Technical supports

However, resources are limited.

Notations

- $[X]$: set $\{1, 2, \ldots, X\}$
- C: number of available courses
- N: number of users
- K: number of steps for negotiation
- $B_{n,k}$: the bundle of user n for step k
- $v_{n,k}$: the valuation of user n for his k-th bundle
- $s_{n,k,c}$: number of enrollments for course c in bundle $B_{n,k}$
- $w_{n,k,c}$: operational cost for course c in bundle $B_{n,k}$
- q_c: enrollment capacity of course c
- $x_{n,k} \in \{0, 1\}$: whether bidder n wins his k-th bundle
- $p_{n,k}$: the price we charge for bidder n's k-th bundle.

Auction Mechanism Design

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Allocation Rule

$$x_{n,k} = \mathcal{A}(B_{n,k}, v_{n,k}, \mathcal{R}) = \begin{cases} 1 & \text{Accept} \\ 0 & \text{Reject} \end{cases} \quad \forall k \in [K], n \in [N]$$

Pricing Rule

$$p_{n,k} = \mathcal{P}(B_{n,k}, v_{n,k}, \mathcal{R})$$

Problem Formulation

$$\text{maximize:} \sum_{n\in[N],k\in[K]} (p_{n,k} - d_{n,k} - \sum_{c\in[C]} \omega_{n,k,c}) \cdot x_{n,k} \tag{1}$$

s.t.

$$\sum_{k\in[K]} x_{n,k} \leq 1, \quad \forall n \in [N]; \tag{2a}$$

$$\sum_{k\in[K]} \sum_{n\in[N]} s_{n,k,c} \cdot x_{n,k} \leq q_c, \quad \forall c \in [C]; \tag{2b}$$

$$x_{n,k} \in \{0,1\}, \quad \forall n \in [N], \forall k \in [K]. \tag{2c}$$

VCG Mechanism [PR03]

Allocation Rule:

$$\begin{aligned} \max \quad & \sum_{n\in[N]} \sum_{k\in[K]} v_{n,k} x_{n,k} \\ \text{s.t.} \quad & \textit{Constraints (2a) - (2c)} \end{aligned}$$

Payment Rule:

$$p_i = \sum_{j\neq i} \sum_{k\in[K]} v_{j,k} \tilde{x}_{j,k} - \sum_{j\neq i} \sum_{k\in[K]} v_{j,k} x_{j,k}$$

where

$$\tilde{x}_{j,k} = \arg\max_{x_{j,k}} \sum_{j\neq i} \sum_{k\in[K]} v_{j,k} x_{j,k}$$

Example - VCG

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Bidding

$$v_{A,\{P\}} = 5, \;\; v_{B,\{Q\}} = 1, \;\; v_{C,\{P,Q\}} = 16$$

Example - VCG

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Formulation

$$\max \quad 5 \cdot x_{A,\{P\}} + x_{B,\{Q\}} + 16 \cdot x_{C,\{P,Q\}} \tag{3}$$

s.t.

$$x_{A,\{P\}} + x_{C,\{P,Q\}} \leq 1 \tag{4a}$$

$$x_{B,\{Q\}} + x_{C,\{P,Q\}} \leq 1 \tag{4b}$$

$$x_{A,\{P\}},\ x_{B,\{Q\}},\ x_{C,\{P,Q\}} \in \{0,1\} \tag{4c}$$

Allocation

$$x_{A,\{P\}} = x_{B,\{Q\}} = 0,\ \ x_{C,\{P,Q\}} = 1$$

Example - VCG

Formulation without user C

$$\max \quad 5 \cdot x_{A,\{P\}} + x_{B,\{Q\}} \tag{5}$$

s.t.

$$x_{A,\{P\}} \leq 1 \tag{6a}$$

$$x_{B,\{Q\}} \leq 1 \tag{6b}$$

$$x_{A,\{P\}},\ x_{B,\{Q\}} \in \{0,1\} \tag{6c}$$

Allocation without C

$$\tilde{x}_{A,\{P\}} = \tilde{x}_{B,\{Q\}} = 1$$

Example - VCG

Pricing

$$
\begin{aligned}
P_c &= \left(\tilde{x}_{A,\{P\}} \cdot v_{A,\{P\}} + \tilde{x}_{B,\{Q\}} \cdot v_{B,\{Q\}}\right) \\
&- \left(x_{A,\{P\}} \cdot v_{A,\{P\}} + x_{B,\{Q\}} \cdot v_{B,\{Q\}}\right) \\
&= (1 \cdot 5 + 1 \cdot 1) - (0 \cdot 5 + 0 \cdot 1) \\
&= 6
\end{aligned}
$$

Virtual Valuation Mechanism [LS04]

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Allocation Rule:

$$\max \quad \sum_{n\in[N]} \sum_{k\in[K]} (\mu_n v_{n,k} x_{n,k} + \lambda_{n,k} x_{n,k})$$
$$\text{s.t.} \quad \textit{Constraints (2a) - (2c)}$$

where μ are positive, $\lambda_{n,k}$ is for particular bidder n and bundle k.
For example, to ensure bidder n never gets bundle k for a price below p_0, set $\lambda_{n,k} = -p_0$.

Virtual Valuation Mechanism

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Payment Rule:

$$p_i = \frac{1}{\mu_i}\left(\sum_{j\neq i}\sum_{k\in[K]}(\mu_j v_{j,k}\tilde{x}_{j,k} + \lambda_{j,k}\tilde{x}_{j,k} - \mu_j v_{j,k}x_{j,k} - \lambda_{j,k}x_{j,k})\right) - \frac{1}{\mu_i}\sum_{k\in[K]}\lambda_{i,k}x_{i,k}$$

where

$$\tilde{x}_{j,k} = \arg\max_{x_{j,k}}\left(\sum_{j\neq i}\sum_{k\in[K]}\mu_j v_{j,k}x_{j,k} + \lambda_{j,k}x_{j,k}\right)$$

Example - VVCA

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Bidding

$$v_{A,\{P\}} = 5, \; v_{B,\{Q\}} = 1, \; v_{C,\{P,Q\}} = 16$$

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Example - VVCA

Formulation

We assign the following λ, μ:

$$\mu_C = 0.5, \;\; \lambda_{B,\{Q\}} = 1$$

Now the integer programming would become:

$$\begin{aligned} \text{max} \quad & 5 \cdot x_{A,\{P\}} + x_{B,\{Q\}} + x_{B,\{Q\}} + 0.5 \cdot 16 \cdot x_{C,\{P,Q\}} \\ \text{s.t.} \quad & \textit{Constraints (4a) - (4c)} \end{aligned}$$

Allocation

$$x_{A,\{P\}} = x_{B,\{Q\}} = 0, \;\; x_{C,\{P,Q\}} = 1$$

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Example - VVCA

Formulation Without C

Without the presence of C, we have:

$$\begin{aligned} \max \quad & 5 \cdot x_{A,\{P\}} + x_{B,\{Q\}} + x_{B,\{Q\}} \\ \text{s.t.} \quad & \textit{Constraints (6a) - (6c)} \end{aligned}$$

Allocation without C

$$\tilde{x}_{A,\{P\}} = \tilde{x}_{B,\{Q\}} = 1$$

Example - VVCA

Pricing

$$\begin{aligned} p'_C &= \frac{1}{\mu_C}\left(\tilde{x}_{A,\{P\}} \cdot v_{A,\{P\}} + \tilde{x}_{B,\{Q\}} \cdot v_{B,\{Q\}} + \lambda_{B,\{Q\}}\tilde{x}_{B,\{Q\}} \cdot v_{B,\right. \\ &- \frac{1}{\mu_C}\left(x_{A,\{P\}} \cdot v_{A,\{P\}} + x_{B,\{Q\}} \cdot v_{B,\{Q\}}\lambda_{B,\{Q\}}x_{B,\{Q\}} \cdot v_{B,\{Q\}}\right. \\ &= \frac{1}{0.5}(1 \cdot 5 + 1 \cdot 1 + 1 \cdot 1 \cdot 1) - \frac{1}{0.5}(0 \cdot 5 + 0 \cdot 1 + 1 \cdot 0 \cdot 1) \\ &= 14 \end{aligned}$$

Thus the revenue of VVCA mechanism would be 14, which is much higher than the revenue of VCG mechanism, i.e., 6.

Business Process in MOOC Industry

Algorithm 1: Negotiation between user n and the platform

```
Initialization: Set t = 1 and flag = 0. Suppose the current
  status of resource capacity is R.
while t ≤ T do
    (a) User n submits his bids (B_{n,k}, v_{n,k}) to the platform.
    (b) The platform calculates x_{n,k} and p_{n,k}, and sends the
        response message to the user.
    (c) If accepted, then the negotiation succeeds, update R,
        set flag = 1, and break. Else (i.e. rejected) the
        negotiation continues with t = t + 1.
end
If flag = 0, then the negotiation fails.
```

iBundle [PU00]

- maintain *ask prices* and *provisional allocation*
- bid is competitive if it is not lower than *ask price*
- bidder is competitive if he has at least one competitive bids

Algorithm

- for each round, bidders submit bids on bundles
- *provisional allocation* computed to maximize seller's revenue
- terminate if each competitive bidder receives a bundle in the provisional allocation
- o.w., *ask prices* are increased by a preset parameter, feedbacks are provided to bidders
- on termination, *provisional allocation* becomes the final allocation, the bidders pay their final bid prices.

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What we have done

- Model formulation for SPOC services
- Mechanisms for Offline combinatorial auction
- Mechanisms for Online combinatorial auction

Future Work

- Compare different mechanisms by simulation
- Real Data Analysis of SPOC Services

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Thanks!