Two-Scale Stochastic Control for Smart-Grid Powered Coordinated Multi-Point Systems

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***Abstract*—In this paper, a novel two-scale stochastic control framework is put forth for smart-grid powered coordinated multi-point (CoMP) systems. Taking into account renewable energy sources (RES), dynamic pricing, two-way energy trading facilities and imperfect energy storage devices, the energy management task is formulated as an infinite-horizon optimization problem minimizing the time-averaged energy transaction cost, subject to the users' quality of service (QoS) requirements. Leveraging the Lyapunov optimization approach and the stochastic subgradient method, a two-scale online control (TS-OC) approach is developed to make online control decisions at two timescales. It is analytically established that the TS-OC is capable of yielding a feasible and asymptotically near-optimal solution.**

Keywords: Two-scale control, battery degeneration, CoMP systems, smart grids, Lyapunov optimization.

I. INTRODUCTION

With ever increasing demand for energy-efficient transmissions, coordinated multi-point processing (CoMP) has been proposed as a promising paradigm for efficient inter-cell interference management in heterogeneous networks (HetNets). In CoMP systems, base stations (BSs) are partitioned into clusters, where BSs per cluster perform coordinated beamforming to serve the users [1]. As the number of BSs in HetNets increases, their electricity consumption constitutes a considerable portion of the operational expenditure of cellular networks, and the global *carbon footprint* [2]. In this context, energy-efficient communication solutions are advocated for their economic and ecological merits [1]–[3]. While BSs considered therein are persistently powered by conventional generators, the current grid infrastructure is on the verge of a major paradigm shift, migrating from the aging grid to a "smart" one.

A few recent works have considered the smart-grid powered CoMP transmissions [4]–[7]. Assuming that the energy harvested from renewable energy sources (RES) is accurately available *a priori*, [4] and [5] considered the energy-efficient resource allocation for RES-powered CoMP downlinks. Building on realistic models, our last work dealt with robust energy management and transmit-beamforming designs for CoMP downlinks [6]. Leveraging novel stochastic optimization tools, we further developed an efficient approach to obtain a feasible and asymptotically optimal online control scheme for smart-grid powered CoMP systems [7].

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A salient assumption in [4]–[7] is that all involved resource allocation tasks are performed in a single timescale. However, RES and wireless channel dynamics typically evolve over different timescales in practice. Extending the traditional Lyapunov optimization approach, [8] introduced a two-scale control algorithm that makes distributed routing and server management decisions to reduce power cost for large-scale data centers. Based on a similar approach, [9] developed an efficient MultiGreen algorithm for data centers with RES, which allows cloud service providers to make online energy purchase decisions at two timescales for minimum cost.

In the present paper, we develop a two-scale online control (TS-OC) approach for smart-grid powered CoMP systems considering RES, dynamic pricing, two-way energy trading facilities and imperfect energy storage devices. Suppose that the RES harvesting occurs at the BSs over a slow timescale relative to the coherence time of wireless channels. The proposed scheme performs an ahead-of-time energy planning upon RES arrivals, while deciding real-time energy balancing and transmit-beamforming schedules per channel coherence time slot. Generalizing the Lyapunov optimization techniques in [8]–[10], we propose a synergetic framework to design and analyze such a two-scale dynamic management scheme to minimize the long-term time-averaged energy transaction cost of the CoMP transmissions, without knowing the distributions of the underlying randomness. Using only historical data, a novel stochastic subgradient approach is proposed to solve the energy planning (sub-)problem, which enjoys a provable near-optimality and faster convergence compared to the empirical-pdf based approach in [8,9]. Rigorous analysis is presented to justify the feasibility and quantify the optimality gap for the proposed two-scale online control algorithm.

The rest of the paper is organized as follows. The system models are described in Section II. The proposed dynamic resource management scheme is developed in Section III. Analysis of the algorithm performance is the subject of Section IV. Numerical tests are provided in Section V, followed by conclusions in Section VI.

II. SYSTEM MODELS

Consider a cluster-based CoMP downlink setup, where a set $\mathcal{I} := \{1, \ldots, I\}$ of distributed BSs (e.g., macro/micro/pico BSs) is selected to serve a set $\mathcal{K} := \{1, \ldots, K\}$ of mobile users, as in e.g., [6,7]. Each BS is equipped with $M \geq 1$ transmit antennas, whereas each user has a single receive antenna. Powered by a smart microgrid, each BS is equipped with one or more energy harvesting devices (solar panels and/or wind turbines), and can perform two-way energy trading with the main grid. In addition, the BSs have batteries so that they can store part of the harvested energy for later use.

As the RES and wireless channel dynamics emerge typically at different timescales in practice, we propose a two-scale control mechanism. Specifically, time is divided in slots of length smaller than the coherence time of the wireless channels; meanwhile, we define the (virtual) "coarse-grained" time intervals in accordance with the slow RES harvesting scale, with each interval consisting of T time slots.

A. Ahead-of-Time Energy Planning

At the beginning of each "coarse-grained" interval ($t = nT$, $n = 1, 2, \ldots$), let $A_i[n]$ denote the RES amount collected per BS $i \in \mathcal{I}$, and $\boldsymbol{a}^n := [A_{1,n}, \ldots, A_i[n]]'$. With $\boldsymbol{a}^n$ available, an energy planner at the central unity decides the energy amounts $E_i[n]$, $\forall i$, to be used in the next T slots per BS i. Given the requested energy $E_i[n]$ and the harvested energy $A_i[n]$, the shortage energy $[E_i[n]-A_i[n]]^+$ is purchased from the grid for BS i with the ahead-of-time (i.e., long-term) price $\alpha_n^{(lt)}$; or, the surplus energy $[A_i[n] - E_i[n]]^+$ is sold to the grid with price $\beta_n^{(lt)}$ for profit, where $[a]^+ := \max\{a, 0\}$ and $\alpha_n^{(lt)} > \beta_n^{(lt)}$. The transaction cost with BS i for such an energy planning is therefore given by

$$G^{(lt)}(E_i[n]) := \alpha_n^{(lt)}[E_i[n]-A_i[n]]^+ - \beta_n^{(lt)}[A_i[n]-E_i[n]]^+. \quad (1)$$

B. CoMP Downlink Transmissions

Per slot t, let $\mathbf{h}_{ik,t} \in \mathbb{C}^M$ denote the vector channel from BS i to user k, $\forall i \in \mathcal{I}$, $\forall k \in \mathcal{K}$; let $\mathbf{h}_{k,t} := [\mathbf{h}'_{1k,t}, \ldots, \mathbf{h}'_{Ik,t}]'$ collect the channel vectors from all BSs to user k, and $\mathbf{H}_t := [\mathbf{h}_{1,t}, \ldots, \mathbf{h}_{K,t}]$. With linear transmit beamforming performed across BSs, the vector signal transmitted to user k is: $\mathbf{q}_k(t) = \mathbf{w}_k(t)s_k(t)$, $\forall k$, where $s_k(t)$ denotes the information-bearing scalar symbol with unit-energy, and $\mathbf{w}_k(t) \in \mathbb{C}^{MI}$ denotes the beamforming vector across the BSs serving user k. The received vector at slot t for user k is therefore $y_k(t) = \mathbf{h}_{k,t}^H \mathbf{q}_k(t) + \sum_{l \neq k} \mathbf{h}_{k,t}^H \mathbf{q}_l(t) + n_k(t)$, where $\mathbf{h}_{k,t}^H \mathbf{q}_k(t)$ is the desired signal of user k, $\sum_{l \neq k} \mathbf{h}_{k,t}^H \mathbf{q}_l(t)$ is the inter-user interference from the same cluster, and $n_k(t)$ denotes additive noise, which is assumed a circularly symmetric complex Gaussian (CSCG) random variable with zero mean and variance σ_k^2.

The signal-to-interference-plus-noise ratio (SINR) at user k can be expressed as

$$\text{SINR}_k(\{\mathbf{w}_k(t)\}) = \frac{|\mathbf{h}_{k,t}^H \mathbf{w}_k(t)|^2}{\sum_{l \neq k}(|\mathbf{h}_{k,t}^H \mathbf{w}_l(t)|^2) + \sigma_k^2} \ . \quad (2)$$

The transmit power at each BS i clearly is given by $P_{x,i}(t) = \sum_{k \in \mathcal{K}} \mathbf{w}_k^H(t) \mathbf{B}_i \mathbf{w}_k(t)$, where the matrix $\mathbf{B}_i := \text{diag}\big(\underbrace{0, \ldots, 0}_{(i-1)M}, \underbrace{1, \ldots, 1}_{M}, \underbrace{0, \ldots, 0}_{(I-i)M}\big) \in \mathbb{R}^{MI \times MI}$ selects the corresponding rows out of $\{\mathbf{w}_k(t)\}_{k \in \mathcal{K}}$ to form the i-th BS's transmit-beamforming vector of size $M \times 1$.

To guarantee QoS per slot user k, it is required that the central controller selects a set of $\{\mathbf{w}_k(t)\}$ satisfying [cf. (2)]

$$\text{SINR}_k(\{\mathbf{w}_k(t)\}) \geq \gamma_k, \quad \forall k \quad (3)$$

where γ_k denotes the target SINR value per user k.

C. Real-Time Energy Balancing

For the i-th BS, the total energy consumption $P_{g,i}(t)$ per slot t includes the transmission-related power $P_{x,i}(t)$, and a constant power $P_c > 0$ due to other components such as data processor, and circuits [5]. We further suppose that $P_{g,i}(t)$ is bounded by $P_g^{\max}$. Namely,

$$P_{g,i}(t) = P_c + \sum_{k \in \mathcal{K}} \mathbf{w}_k^H(t) \mathbf{B}_i \mathbf{w}_k(t) \leq P_g^{\max}, \quad \forall i. \quad (4)$$

Per slot t, the BS i is allowed to perform real-time energy trading with the main grid to balance its supply with demand. Let $P_i(t)$ denote the real-time energy amount that is purchased from ($P_i(t) > 0$) or sold to ($P_i(t) < 0$) the grid by BS i. Let $\alpha_t^{(rt)}$ and $\beta_t^{(rt)}$ ($\alpha_t^{(rt)} > \beta_t^{(rt)}$) denote the real-time energy purchase and selling prices, respectively. Then the real-time energy transaction cost for BS i is

$$G^{(rt)}(P_i(t)) := \alpha_t^{(rt)}[P_i(t)]^+ - \beta_t^{(rt)}[-P_i(t)]^+. \quad (5)$$

D. Energy Storage with Degeneration

For the battery of the i-th BS, let $C_i(0)$ denote the initial amount of stored energy, and $C_i(t)$ its state of charge (SoC) at the beginning of time slot t. The battery capacity is assumed bounded by $C^{\min}$ and $C^{\max}$. With $P_{b,i}(t)$ denoting the energy delivered to or drawn from the battery at slot t, the stored energy then obeys the dynamic equation

$$C_i(t+1) = \eta C_i(t) + P_{b,i}(t), \ C^{\min} \leq C_i(t) \leq C^{\max}, \ \forall i \quad (6)$$

where $\eta \in (0, 1]$ denotes the storage efficiency.

The amount of power (dis)charged is assumed bounded by

$$P_b^{\min} \leq P_{b,i}(t) \leq P_b^{\max}, \quad \forall i. \quad (7)$$

With $n_t := \lfloor \frac{t}{T} \rfloor$ and consideration of $P_{b,i}(t)$, we have the following demand-and-supply balance equation per slot t:

$$P_c + \sum_{k \in \mathcal{K}} \mathbf{w}_k^H(t) \mathbf{B}_i \mathbf{w}_k(t) + P_{b,i}(t) = \frac{E_i[n_t]}{T} + P_i(t), \ \forall i. \quad (8)$$

III. Dynamic Resource Management Scheme

Note that the smart-grid powered CoMP downlink to be controlled is a stochastic system. The goal is to design an online resource management scheme that chooses the ahead-of-time energy-trading amounts $\{E_i[n], \forall i\}$ at every $t = nT$, as well as the real-time energy-trading amounts $\{P_i(t), \forall i\}$, battery (dis)charging amounts $\{P_{b,i}(t), \forall i\}$, and the CoMP beamforming vectors $\{\mathbf{w}_k(t), \forall k\}$ per slot t, so as to minimize the expected total energy transaction cost, without knowing the distributions of the underlying random processes.

According to (1) and (5), define the energy transaction cost for BS i per slot t as:

$$\Phi_i(t) := \frac{1}{T} G^{(lt)}(E_i[n_t]) + G^{(rt)}(P_i(t)). \tag{9}$$

Let $\mathcal{X} := \{E_i[n], \forall i, n; P_i(t), P_{b,i}(t), C_i(t), \forall i, t; \mathbf{w}_k(t), \forall k, t\}$. The problem of interest is to find

$$\begin{aligned}\Phi^{opt} := \min_{\mathcal{X}} \; & \lim_{N\to\infty} \frac{1}{NT} \sum_{t=0}^{NT-1} \sum_{i\in\mathcal{I}} \mathbb{E}\{\Phi_i(t)\} \\ \text{s. t.} \quad & (3), (4), (6), (7), (8), \quad \forall t.\end{aligned} \tag{10}$$

A. Two-Scale Online Control Algorithm

Problem (10) is a stochastic optimization task. We next generalize and integrate the Lyapunov optimization techniques in [8,10] to develop a TS-OC algorithm. To start, we assume the following two relatively mild conditions for the system:

$$P_b^{\max} \geq (1-\eta) C^{\min} \tag{11}$$

$$C^{\max} - C^{\min} \geq \frac{1-\eta^T}{1-\eta}(P_b^{\max} - P_b^{\min}). \tag{12}$$

Condition (11) simply implies that the energy leakage of the battery can be compensated by the charging. Condition (12) requires that the allowable SoC range is large enough to accommodate the largest possible charging/discharging over T time slots of each coarse-grained interval.

Our algorithm depends on two parameters, namely a "queue perturbation" parameter Γ, and a weight parameter V. Define $\bar{\alpha} := \max\{\alpha_t^{(rt)}, \forall t\}$ and $\underline{\beta} := \min\{\beta_t^{(rt)}, \forall t\}$. Derived from the feasibility requirement of the proposed algorithm, any pair (Γ, V) that satisfies the following conditions can be used:

$$\Gamma^{\min} \leq \Gamma \leq \Gamma^{\max}, \quad 0 < V \leq V^{\max} \tag{13}$$

where

$$\Gamma^{\min} := \max_{\tau=1,\dots,T} \left\{ \frac{1}{\eta^\tau} (\frac{1-\eta^\tau}{1-\eta} P_b^{\max} - C^{\max}) - V\underline{\beta} \right\} \tag{14}$$

$$\Gamma^{\max} := \min_{\tau=1,\dots,T} \left\{ \frac{1}{\eta^\tau} (\frac{1-\eta^\tau}{1-\eta} P_b^{\min} - C^{\min}) - V\bar{\alpha} \right\} \tag{15}$$

$$V^{\max} := \min_{\tau=1,\dots,T} \left\{ \frac{C^{\max} - C^{\min} - \frac{1-\eta^\tau}{1-\eta}(P_b^{\max} - P_b^{\min})}{\eta^\tau (\bar{\alpha} - \underline{\beta})} \right\}. \tag{16}$$

We now present the proposed TS-OC algorithm:

- **Initialization**: Select Γ and V, and introduce a virtual queue $Q_i(t) := C_i(t) + \Gamma$, $\forall i$.
- **Energy planning**: Per interval $\tau = nT$, with $\boldsymbol{\zeta}_n := \{\boldsymbol{a}^n, \alpha_n^{(lt)}, \beta_n^{(lt)}\}$ available, determine the energy amounts $\{E_i^*[n], \forall i\}$ by solving

$$\begin{aligned}\min \sum_{i\in\mathcal{I}} \Bigg\{ V\Big[G^{(lt)}(E_i[n]) + \sum_{t=\tau}^{\tau+T-1} \mathbb{E}\{G^{(rt)}(P_i(t))\} \Big] \\ + \sum_{t=\tau}^{\tau+T-1} Q_i(\tau) \mathbb{E}\{P_{b,i}(t)\} \Bigg\} \\ \text{s. t. } (3), (4), (7), (8), \; \forall t = \tau, \dots, \tau+T-1\end{aligned} \tag{17}$$

where expectations are taken over $\boldsymbol{\xi}_t := \{\alpha_t^{(rt)}, \beta_t^{(rt)}, \mathbf{H}_t\}$. Then the BSs trade energy with the main grid to supply an average amount $E_i^*[n]/T$ per slot $t = \tau, \dots, \tau+T-1$.
- **Energy balancing and beamforming schedule**: At every slot $t \in [nT, (n+1)T-1]$, with $E_i[n] = E_i^*[n]$ determined and $\boldsymbol{\xi}_t$ available, decide $\{P_i^*(t), P_{b,i}^*(t), \forall i; \mathbf{w}_k^*(t), \forall k\}$ by solving

$$\begin{aligned}\min \sum_{i\in\mathcal{I}} \Big\{ VG^{(rt)}(P_i(t)) + Q_i(nT) P_{b,i}(t) \Big\} \\ \text{s. t.} \quad (3), (4), (7), (8).\end{aligned} \tag{18}$$

  The BSs perform real-time energy trading with the main grid based on $\{P_i^*(t), \forall i\}$, and coordinated beamforming based on $\{\mathbf{w}_k^*(t), \forall k\}$.
- **Queue updates**: Per slot t, charge (or discharge) the battery based on $\{P_{b,i}^*(t)\}$, so that the stored energy follows $C_i(t+1) = \eta C_i(t) + P_{b,i}^*(t)$, $\forall i$; and update the virtual queues $Q_i(t)$, $\forall i$, accordingly.

Next, we develop efficient solvers of (17) and (18) to obtain the TS-OC algorithm.

B. Real-Time Energy Balancing and Beamforming

It is easy to argue that the objective (18) is convex. Indeed, with $\alpha_t^{(rt)} > \beta_t^{(rt)}$, the transaction cost with $P_i(t)$ can be alternatively written as $G^{(rt)}(P_i(t)) = \max\{\alpha_t^{(rt)} P_i(t), \; \beta_t^{(rt)} P_i(t)\}$, which is clearly convex [11]; and so is the objective in (18).

By proper rearrangement, the SINR constraints in (3) can be rewritten to convex second-order cone (SOC) constraints [12]; that is,

$$\begin{aligned}&\sqrt{\sum_{l\neq k} |\boldsymbol{h}_{k,t}^H \boldsymbol{w}_l(t)|^2 + \sigma_k^2} \leq \frac{1}{\sqrt{\gamma_k}} \mathrm{Re}\{\boldsymbol{h}_{k,t}^H \boldsymbol{w}_k(t)\}, \\ &\mathrm{Im}\{\boldsymbol{h}_{k,t}^H \boldsymbol{w}_k(t)\} = 0, \quad \forall k.\end{aligned} \tag{19}$$

We can then rewrite the problem (18) as

$$\begin{aligned}\min \sum_{i\in\mathcal{I}} \Bigg\{ VG^{(rt)}(P_c + \sum_{k\in\mathcal{K}} \mathbf{w}_k^H(t) \mathbf{B}_i \mathbf{w}_k(t) + P_{b,i}(t) - \frac{E_i^*[n_t]}{T}) \\ + Q_i(n_t T) P_{b,i}(t) \Bigg\} \quad \text{s. t. } (4), (7), (19).\end{aligned} \tag{20}$$

As $G^{(rt)}(\cdot)$ is convex and increasing, it is easy to see that $G^{(rt)}(P_c + \sum_k \mathbf{w}_k^H(t)\mathbf{B}_i\mathbf{w}_k(t) + P_{b,i}(t) - \frac{E_i^*[n_t]}{T})$ is jointly convex in $(P_{b,i}(t), \{\mathbf{w}_k(t)\})$ [11, Sec. 3.2.4]. It then readily follows that (20) is a convex optimization problem, which can be solved by general interior-point solvers.

C. Ahead-of-Time Energy Planning

To solve (17), here we propose a stochastic gradient approach. Suppose that $\boldsymbol{\xi}_t$ is i.i.d. across time slots. For stationary $\boldsymbol{\xi}_t$, we can remove the index t from all optimization variables, and rewrite (17) as (with short-hand notation $Q_i[n] := Q_i(nT)$)

$$\min \sum_{i\in\mathcal{I}}\Big\{VG^{(lt)}(E_i[n])+T\mathbb{E}\left[VG^{(rt)}(P_i(\boldsymbol{\xi}_t))+Q_i[n]P_{b,i}(\boldsymbol{\xi}_t)\right]\Big\}$$

$$\text{s. t.} \sqrt{\sum_{l\neq k}|\boldsymbol{h}_k^H\boldsymbol{w}_l(\boldsymbol{\xi}_t)|^2+\sigma_k^2} \leq \frac{1}{\sqrt{\gamma_k}}\text{Re}\{\boldsymbol{h}_k^H\boldsymbol{w}_k(\boldsymbol{\xi}_t)\},$$

$$\text{Im}\{\boldsymbol{h}_k^H\boldsymbol{w}_k(\boldsymbol{\xi}_t)\}=0,\ \forall k,\forall\boldsymbol{\xi}_t \tag{21a}$$

$$P_b^{\min} \leq P_{b,i}(\boldsymbol{\xi}_t) \leq P_b^{\max},\ \forall i,\forall\boldsymbol{\xi}_t \tag{21b}$$

$$P_c+\sum_{k\in\mathcal{K}}\mathbf{w}_k^H(\boldsymbol{\xi}_t)\mathbf{B}_i\mathbf{w}_k(\boldsymbol{\xi}_t) \leq P_g^{\max},\ \forall i,\forall\boldsymbol{\xi}_t \tag{21c}$$

$$P_c+\sum_{k\in\mathcal{K}}\mathbf{w}_k^H(\boldsymbol{\xi}_t)\mathbf{B}_i\mathbf{w}_k(\boldsymbol{\xi}_t)+P_{b,i}(\boldsymbol{\xi}_t) = \frac{E_i[n]}{T}+P_i(\boldsymbol{\xi}_t),\ \forall i,\forall\boldsymbol{\xi}_t. \tag{21d}$$

Since the energy planning problem (17) only determines the optimal ahead-of-time energy purchase $E_i^*[n]$, we can then eliminate the variable P_i and write (21) as an unconstrained optimization problem with respect to the variable $E_i^*[n]$, namely

$$\min_{\{E_i^*[n]\}} \sum_{i\in\mathcal{I}}[VG^{(lt)}(E_i[n])+T\bar{G}^{(rt)}(\{E_i[n]\})] \tag{22}$$

where we define

$$\bar{G}^{(rt)}(\{E_i[n]\}) := \min\sum_{i\in\mathcal{I}}\mathbb{E}\Big\{V\Psi^{(rt)}(E_i[n],P_{b,i}(\boldsymbol{\xi}_t),\{\mathbf{w}_k(\boldsymbol{\xi}_t)\}) +Q_i[n]P_{b,i}(\boldsymbol{\xi}_t)\Big\} \quad \text{s. t.} \quad \text{(21a), (21b), (21c)} \tag{23}$$

with the compact notation

$$\Psi^{(rt)}(E_i,P_{b,i},\{\mathbf{w}_k\}) := G^{(rt)}(P_c+\sum_{k\in\mathcal{K}}\mathbf{w}_k^H\mathbf{B}_i\mathbf{w}_k+P_{b,i}-\frac{E_i}{T}).$$

It can been observed that (22) is generally a nonsmooth and unconstrained convex problem with respect to $\{E_i[n]\}$, which can be solved using the stochastic subgradient iteration described next.

The subgradient of $G^{(lt)}(E_i[n])$ can be first written as

$$\partial G^{(lt)}(E_i[n]) = \begin{cases}\alpha_n^{(lt)}, & \text{if } E_i[n] > A_i[n]\\ \beta_n^{(lt)}, & \text{if } E_i[n] < A_i[n]\\ \text{any } x\in[\beta_n^{(lt)},\alpha_n^{(lt)}], & \text{if } E_i[n]=A_i[n].\end{cases}$$

With $\{P_{b,i}^E(\boldsymbol{\xi}_t),\mathbf{w}_k^E(\boldsymbol{\xi}_t)\}$ denoting the optimal solution for the problem in (23), the partial subgradient of $\bar{G}^{(rt)}(\{E_i[n]\})$ with respect to $E_i[n]$ is $\partial_i\bar{G}^{(rt)}(\{E_i[n]\}) = V\mathbb{E}\{\partial\Psi^{(rt)}(E_i[n],P_{b,i}^E(\boldsymbol{\xi}_t),\{\mathbf{w}_k^E(\boldsymbol{\xi}_t)\})\}$, where

$$\partial\Psi^{(rt)}(E_i[n],P_{b,i}^E(\boldsymbol{\xi}_t),\{\mathbf{w}_k^E(\boldsymbol{\xi}_t)\}) = \begin{cases}\frac{-\beta_t^{(rt)}}{T}, \text{ if } \frac{E_i[n]}{T} > \Delta\\ \frac{-\alpha_t^{(rt)}}{T}, \text{ if } \frac{E_i[n]}{T} < \Delta\\ x\in[\frac{-\alpha_t^{(rt)}}{T},\frac{-\beta_t^{(rt)}}{T}], \text{else}\end{cases}$$

with $\Delta := P_c+\sum_k {\mathbf{w}_k^E}^H(\boldsymbol{\xi}_t)\mathbf{B}_i\mathbf{w}_k^E(\boldsymbol{\xi}_t)+P_{b,i}^E(\boldsymbol{\xi}_t)$.

Defining $\bar{g}_i(E_i) := V\partial G^{(lt)}(E_i)+T\partial_i\bar{G}^{(rt)}(\{E_i\})$, a standard subgradient descent iteration can be employed to find the optimal $E_i^*[n]$ for (22), as (j denotes iteration index)

$$E_i^{(j+1)}[n] = [E_i^{(j)}[n]-\mu^{(j)}\bar{g}_i(E_i^{(j)}[n])]^+, \quad \forall i \tag{24}$$

where $\{\mu^{(j)}\}$ is the sequence of stepsizes.

Implementing (24) essentially requires performing (high-dimensional) integration over the unknown multivariate distribution function of $\boldsymbol{\xi}_t$ present in $\bar{g}_i$ through $\bar{G}^{(rt)}$ in (23). To circumvent this impasse, a stochastic subgradient approach is devised based on the past realizations $\{\boldsymbol{\xi}_\tau,\tau=0,1,\ldots,nT-1\}$. Per iteration j, we randomly draw a realization $\boldsymbol{\xi}_\tau$ from past realizations, and run the following iteration

$$E_i^{(j+1)}[n] = [E_i^{(j)}[n]-\mu^{(j)}g_i(E_i^{(j)}[n])]^+, \quad \forall i \tag{25}$$

where $g_i(E_i^{(j)}[n]) := V(\partial G^{(lt)}(E_i^{(j)}[n])+T\partial\Psi^{(rt)}(E_i^{(j)}[n], P_{b,i}^E(\boldsymbol{\xi}_\tau),\{\mathbf{w}_k^E(\boldsymbol{\xi}_\tau)\}))$ with $\{P_{b,i}^E(\boldsymbol{\xi}_\tau),\mathbf{w}_k^E(\boldsymbol{\xi}_\tau)\}$ obtained by solving a convex problem (23) with $E_i[n]=E_i^{(j)}[n]$.

IV. Performance Analysis

In this section, we show that the TS-OC can yield a feasible and asymptotically (near-)optimal solution for problem (10).

A. Feasibility Guarantee

Note that the constraints in (6) are ignored in problems (17) and (18). Yet, we will show that by selecting a pair (Γ,V) in (13), we can guarantee that $C^{\min} \leq C_i(t) \leq C^{\max}$, $\forall i,t$; meaning, the online control policy produced by the TS-OC is feasible.

To this end, we first show the following lemma.

Lemma 1: If $\bar{\alpha} := \max\{\alpha_t^{(rt)},\forall t\}$ and $\underline{\beta} := \min\{\beta_t^{(rt)},\forall t\}$, the battery (dis)charging amounts $P_{b,i}^*(t)$ obtained from the TS-OC algorithm satisfy: i) $P_{b,i}^*(t)=P_b^{\min}$, if $C_i(n_tT) > -V\underline{\beta}-\Gamma$; and ii) $P_{b,i}^*(t)=P_b^{\max}$, if $C_i(n_tT) < -V\bar{\alpha}-\Gamma$.

Lemma 1 reveals partial characteristics of the dynamic TS-OC policy. Specifically, the battery must be fully discharged ($P_{b,i}^*(t)=P_b^{\min}$) when the energy queue (i.e., battery SoC) is large enough, and fully charged ($P_{b,i}^*(t)=P_b^{\max}$) when the energy queue is small enough.

Based on the structure in Lemma 1, we can thus establish the following result.

Proposition 1: Under the conditions (11)–(12), the TS-OC algorithm with any pair (Γ,V) specified in (13) guarantees $C^{\min} \leq C_i(t) \leq C^{\max}$, $\forall i,t$.

B. Asymptotic Optimality

Define $\bar{C}_i := \frac{1}{NT}\sum_{t=0}^{NT-1}\mathbb{E}\{C_i(t)\}$ and $\bar{P}_{b,i} := \frac{1}{NT}\sum_{t=0}^{NT-1}\mathbb{E}\{P_{b,i}(t)\}$. Since $P_{b,i}(t)\in[P_b^{\min},P_b^{\max}]$ and $C_i(t+1)=\eta C_i(t)+P_{b,i}(t)$, it holds that

$$\bar{P}_{b,i} = \frac{1}{NT}\sum_{t=0}^{NT-1}\mathbb{E}\{C_i(t+1)-\eta C_i(t)\} = (1-\eta)\bar{C}_i. \tag{26}$$

The proofs for all lemmas and propositions are omitted due to limited space, and can be found in the extended journal version [13].

As $C_i(t) \in [C^{\min}, C^{\max}]$, $\forall t$, (26) then implies

$$(1-\eta)C^{\min} \le \bar{P}_{b,i} \le (1-\eta)C^{\max}, \quad \forall i. \tag{27}$$

Consider now the following problem

$$\begin{aligned} \tilde{\Phi}^{opt} := \min_{\mathcal{X}} \ & \lim_{N\to\infty} \frac{1}{NT} \sum_{t=0}^{NT-1} \sum_{i\in\mathcal{I}} \mathbb{E}\{\Phi_i(t)\} \\ \text{s. t.} \quad & (3), (4), (7), (8), (27), \ \forall t. \end{aligned} \tag{28}$$

Note that the constraints in (6), $\forall t$, are replaced by (27). The problem (28) is thus a relaxed version of (10) [10]. Specifically, any feasible solution of (10) also satisfies (28); that is, $\tilde{\Phi}^{opt} \le \Phi^{opt}$.

As the variables are "decoupled" across time slots, this problem has an easy-to-characterize stationary optimal control policy as formally stated in the next lemma.

Lemma 2: If $\boldsymbol{\zeta}_n$ and $\boldsymbol{\xi}_t$ are i.i.d., there exists a stationary control policy $\mathcal{P}^{stat}$ that is a pure (possibly randomized) function of the current $(\boldsymbol{\zeta}_{n_t}, \boldsymbol{\xi}_t)$, while satisfying (3), (4), (7), (8), and providing the following guarantees per t:

$$\begin{aligned} & \mathbb{E}\{\sum_{i\in\mathcal{I}} \Phi_i^{stat}(t)\} = \tilde{\Phi}^{opt} \\ & (1-\eta)C^{\min} \le \mathbb{E}\{P_{b,i}^{stat}(t)\} \le (1-\eta)C^{\max}, \ \ \forall i \end{aligned} \tag{29}$$

where $P_{b,i}^{stat}(t)$ denotes the decided (dis)charging amount, $\Phi_i^{stat}(t)$ the resultant transaction cost by policy $\mathcal{P}^{stat}$.

Lemma 2 plays a critical role in establishing the following result.

Proposition 2: Suppose that conditions (11)–(13) hold. If $\boldsymbol{\zeta}_n$ and $\boldsymbol{\xi}_t$ are i.i.d. across time, then the time-averaged cost under the proposed TS-OC algorithm satisfies

$$\lim_{N\to\infty} \frac{1}{NT} \sum_{t=0}^{NT-1} \sum_{i\in\mathcal{I}} \mathbb{E}\{\Phi_i^*(t)\} \le \Phi^{opt} + \frac{M_1 + M_2 + M_3}{V}$$

where the constants $M_1 := \frac{IT(1-\eta)}{2\eta(1-\eta^T)} M_B$, $M_2 := \frac{I[T(1-\eta)-(1-\eta^T)]}{(1-\eta)(1-\eta^T)} M_B$, and $M_3 := I(1-\eta)M_C$, with $M_B := \max\{[(1-\eta)\Gamma + P_b^{\min}]^2, [(1-\eta)\Gamma + P_b^{\max}]^2\}$ and $M_C := \max\{(\Gamma + C^{\min})^2, (\Gamma + C^{\max})^2\}$; $\Phi_i^*(t)$ denotes the resultant cost with the TS-OC, and Φ^{opt} is the optimal value of (10) under any feasible control algorithm, including the one knowing all future realizations.

Proposition 2 asserts that the proposed TS-OC algorithm yields a time-averaged cost with optimality gap smaller than $(M_1 + M_2 + M_3)/V$. Intuitively, the gap M_1/V is inherited from the underlying stochastic subgradient method. The gap M_2/V is introduced by the inaccurate queue lengths in use (since we replace $Q_i(t)$ by $Q_i(nT)$) (since we use $Q_i(nT)$, instead of $Q_i(t)$, for all $t = nT, \ldots, (n+1)T - 1$), while the gap M_3/V is incurred by the battery imperfections.

C. Main Theorem

Based on Propositions 1–2, we are ready to arrive at our main result.

Theorem 1: Suppose that conditions (11)–(13) hold and $(\boldsymbol{\zeta}_n, \boldsymbol{\xi}_t)$ are i.i.d. over slots. Then the proposed TS-OC yields a feasible dynamic control scheme for (10), which is asymptotically near-optimal in the sense that

$$\Phi^{opt} \le \lim_{N\to\infty} \frac{1}{NT} \sum_{t=0}^{NT-1} \sum_{i\in\mathcal{I}} \mathbb{E}\{\Phi_i^*(t)\} \le \Phi^{opt} + \frac{M}{V}$$

where $M := M_1 + M_2 + M_3$, as specified in Proposition 2.

Interesting comments on the minimum optimality gap with the TS-OC are now in order.

1) When $\eta = 1$ (perfect battery), the optimality gap between the TS-OC and the offline optimal scheduling reduces to $M/V = (M_1 + M_2)/V = \frac{IT}{2V} \max\{(P_b^{\min})^2, (P_b^{\max})^2\}$. The asymptotic optimality can be achieved when we have very small price difference $(\bar{\alpha} - \underline{\beta})$, or very large battery capacities $C^{\max}$, so that $V^{\max} \to \infty$.

2) When $\eta \in (0, 1)$, the constants M_1, M_2 and M_3 are in fact functions of Γ. For a given $V^{\max}$, the minimum optimality gap, $G^{\min}(V^{\max})$, can be obtained by solving the following problem:

$$\min_{(V,\Gamma)} \ \frac{M}{V} = \frac{M_1(\Gamma)}{V} + \frac{M_2(\Gamma)}{V} + \frac{M_3(\Gamma)}{V}, \quad \text{s. t.} \ \ (13). \tag{30}$$

Problem (30) can be easily proven convex [11], and can be efficiently solved by general interior-point methods. Note that $G^{\min}(V^{\max})$ no longer monotonically decreases with respect to $V^{\max}$ (or $C^{\max}$); see also [10]. The smallest possible optimality gap can be numerically computed by one dimensional search over $G^{\min}(V^{\max})$ with respect to $V^{\max}$.

V. Numerical Tests

TABLE I
PARAMETER VALUES. THE UNITS ARE KW OR KWH.

P_c	$P_g^{\max}$	$P_b^{\min}$	$P_b^{\max}$	$C^{\min}$	$C^{\max}$	$C_i(0)$
10	50	-2	2	0	80	0

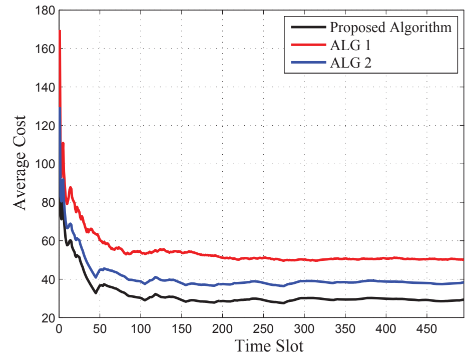

Fig. 1. Comparison of average transaction cost.

The proposed TS-OC was numerically tested on a CoMP network consisting of $I = 2$ BSs each with $M = 2$ transmit antennas, and $K = 3$ mobile users. Each coarse-grained interval consists of $T = 5$ time slots. The limits of $P_{g,i}$, $P_{b,i}$ and C_i, as well as the values of $C_i(0)$ and P_c are listed in Table I. The battery storage efficiency is $\eta = 0.95$. The ahead-of-time and real-time energy purchase prices $\alpha_n^{(lt)}$ and $\alpha_t^{(rt)}$ are generated from folded normal distributions, with $\mathbb{E}\{\alpha_n^{(lt)}\} = 1.15$ and $\mathbb{E}\{\alpha_t^{(rt)}\} = 2.3$. The selling prices

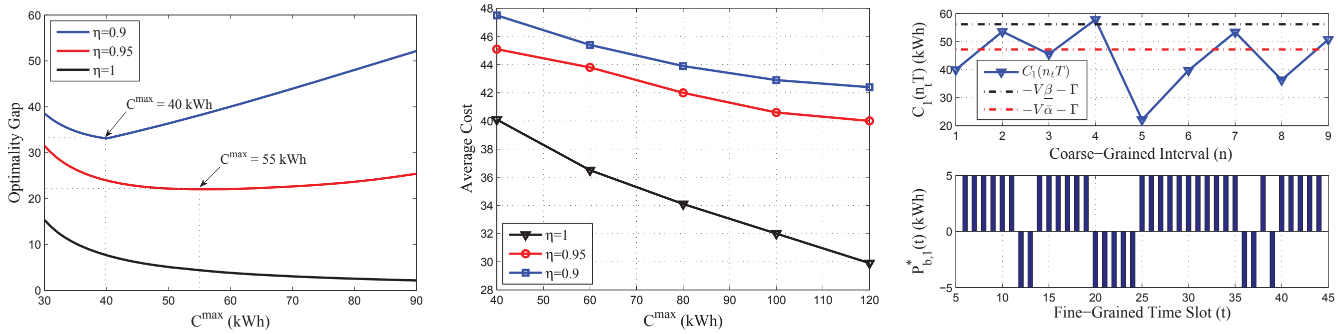

Fig. 2. (*left*): Optimality-gap versus battery capacity $C^{\max}$; (*center*): Average transaction cost versus battery capacity $C^{\max}$; (*right*): TS-OC based schedule of the battery SoC $C_1(n_tT)$ and battery (dis)charging actions $P^*_{b,1}(t)$, where $P_b^{\max} = 5$ kWh and $P_b^{\min} = -5$ kWh.

are set as $\beta_n^{(lt)} = 0.9 \times \alpha_n^{(lt)}$ and $\beta_t^{(rt)} = 0.3 \times \alpha_t^{(rt)}$. The harvested energy $A_i[n]$ is also generated from a folded normal distribution. Finally, the Lyapunov control parameter V is chosen as $V = V^{\max}$.

Two baseline schemes are introduced to benchmark the performance of our TS-OC. ALG 1 is a one-scale scheme without ahead-of-time energy planning; and ALG 2 performs two-scale online control without leveraging neither RES nor energy storage devices. Fig. 1 compares the running-average transaction costs of the proposed algorithm and ALGs 1-2. Within 500 time slots, the proposed approach converges to the lowest transaction cost, while ALGs 1-2 incur about 71.0% and 30.6% larger costs. Intuitively, the TS-OC algorithm intelligently takes advantage of the ahead-of-time energy planning, and the renewable energy and batteries, to hedge against future potential high energy cost.

The theoretical optimality-gaps [cf. (30)] and the average transaction cost of the TS-OC are compared under different battery efficiencies $\eta = 0.9, 0.95, 1$ in Figs. 2(a) and (b), respectively. In Fig. 2(a), the optimality-gap for $\eta = 1$ diminishes as $C^{\max}$ grows as Theorem 1; whereas the gaps for $\eta = 0.9$ and 0.95 first decrease and then increase, reaching the lowest points at $C^{\max} = 40$ and 55 kWh, respectively. As expected, the gap for $\eta = 0.9$ remains the largest across the entire spectrum of battery capacity. In Fig. 2(b), clearly the average costs monotonically decrease as $C^{\max}$ grows. The BSs with imperfect batteries ($\eta = 0.9, 0.95$) require larger budgets for energy purchase than the ones with perfect batteries ($\eta = 1$), thus compensating for the battery degeneration losses.

Taking a deeper look, the battery SoC $C_1(n_tT)$ and the real-time battery (dis)charging $P^*_{b,1}(t)$ are jointly depicted in Fig. 2(c) to reveal the (dis)charging characteristics stated in Lemma 1. It can be observed that the TS-OC dictates the full discharge $P^*_{b,1}(t) = P_b^{\min}$ in the incoming 5 fine-grained slots $t \in [20, 24]$ when $C_1(n_tT) > -V\underline{\beta} - \Gamma$ at $n = 4$, while the battery is fully charged $P^*_{b,1}(t) = P_b^{\max}$ when $C_1(n_tT) < -V\bar{\alpha} - \Gamma$ at $n = 1, 3, 5, 6, 8$.

VI. Conclusions

Two-scale dynamic resource allocation was developed for smart-grid powered CoMP transmissions. A stochastic optimization problem was formulated to minimize the long-term average energy transaction cost. Capitalizing on the Lyapunov optimization technique and the stochastic subgradient iteration, a two-scale online algorithm was proposed to obtain a feasible and asymptotically near-optimal solution 'on the fly.'

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