Market Share Analysis with Brand Effect (Extended Abstract)

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ABSTRACT

We investigate the effect of brand in market competition by proposing a variant Hotelling model, with an additional brand effect term modeled by a function of its market area of the form $-\beta\cdot(\text{Market Area})^q$, where β captures the brand influence and q captures how market share affects the brand. We show that at equilibrium, a company's price is proportional to its market area over the competition intensity with its neighbors, a result that quantitatively reconciles the common belief of a company's pricing power. We also study an interesting "wipe out" phenomenon that only appears when $q > 0$, which is similar to the "undercut" phenomenon in the Hotelling model, where companies may suddenly lose the entire market area with a small price increment. Our results offer novel insight into market pricing and positioning under competition with brand effect.

Keywords

Game theory; Market share; Brand effect

1. INTRODUCTION

Both market area and brand name effect have been subjects with extensive studies, as showed in [2] [4] [7] [6]. However, there are limited works on analyzing the relationship between brand name effect and market share. We use a variant Hotelling model (see [3] [1] [5]) to analyze how brand name and market area affects each others.

2. MODEL AND PRELIMINARIES

Consider an abstract market modeled by the K-dimensional Euclidean space $\mathcal{M} = \mathbb{R}^K$, where each axis represents one feature of the products in consideration. For simplicity, we assume that all companies produce products different only in features we consider, with zero cost and no limit on production capacity. Without loss of generality, we assume that there exists infinitely many companies and they are placed all over the market. Meanwhile, customers are assumed to be uniformly distributed among the space with equal demand of the products. Let $\mathcal{N}$ denotes the set of companies that we consider, all of whose locations $\mathbf{x}_i$ are inside the square of $\{\mathbf{x} \mid \|\mathbf{x}\|_\infty \leq B/2\}$. Companies in $\mathcal{N}$ can decide their own mill prices, while other companies are assumed to have fixed mill prices. On the other hand, customers at $\mathbf{x}$ will choose to buy products from the seller with lowest aggregate price:

$$P_i\{\mathbf{x}\} = P_i + D(\mathbf{x}, \mathbf{x_i}) - \beta S_i^q, \quad \beta, q \geq 0, \quad \forall i \in \mathcal{N}, \qquad (1)$$

where (i)P_i is i's mill price; (ii) $D(\mathbf{x}, \mathbf{x_i})$ is the distance function between $\mathbf{x}$ and $\mathbf{x_i}$; (iii) S_i is i's market area, or market share under normalization, q denotes how the market share contributes to the brand name, and β represents the degree to which customers consider the brand names when making decisions.

A company i's market area is the volume of the market space where all customers inside choose to buy from i, which is proven to be a convex polyhedron. For tractability and without loss of generality, we adopt the commonly used quadratic form distance function between customer and company (see [1]) in (1) here, i.e., $D(\mathbf{x}, \mathbf{x_i}) = ||\mathbf{x} - \mathbf{x_i}||_2^2$.

3. MARKET EQUILIBRIUM WHEN $Q = 0$

We present our results when $q = 0$ in this section.

THEOREM 1. *In the 1D market with $q = 0$, Nash equilibrium always exists, and when market is at equilibrium, we must have:*

$$P_i = \frac{2d_i d_{i-1}}{d_i + d_{i-1}} S_i, \forall i \in \mathcal{N}, \qquad (2)$$

where $d_i = x_{i+1} - x_i$ is the distance between $\mathbf{x}_{i+1}$ and $\mathbf{x}_i$.

The existence of equilibrium is proved by showing the utility function of a company is exactly a parabola. Equation (2) tells us that a company's price at equilibrium is proportional to its market area. The coefficient $\frac{2d_i d_{i-1}}{d_i + d_{i-1}}$ is a constant for a company since their locations are fixed. Denote $\frac{1}{\gamma} = \frac{2d_i d_{i-1}}{d_i + d_{i-1}}$, or $\gamma = \frac{1}{2}(\frac{1}{d_i} + \frac{1}{d_{i-1}})$. Notice that with bigger d_i, d_{i-1} values, γ becomes smaller, which implies higher equilibrium prices with the same market area. Therefore, γ can be viewed as the *competition intensity*, i.e., farther distance between companies mitigates the competition and increase company profit. This is similar to the maximal differentiation principle [1], which states that companies should not choose similar positions in the market, i.e., larger d_i values. The simple form in Theorem 1 that equilibrium price is determined by market area over competition intensity matches our intuition that companies with more market share or less competition in products usually have more pricing power.

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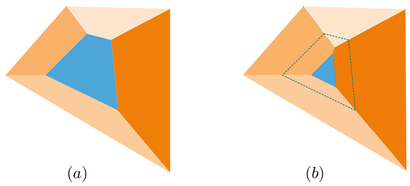

(a) (b)

Figure 1: From (a) to (b), company in blue colour suffers from decrease of market area and neighbors due to price increment.

We now turn to the 2D case. The biggest challenge in analyzing the 2D market is that companies' neighbors may change when prices vary (Figure 1), while in the 1D market, company i's neighbors remain unchaged. Due to the change in neighbors, each company's utility function will be piecewise continuous (Figure 2), i.e., it changes every time a neighbor comes or goes. Moreover, since companies' locations are arbitrary, the shape of a company's market area may be irregular, which makes the analysis more difficult, even proving existence of equilibrium is non trivial.

THEOREM 2. *In the 2D market with $q = 0$, Nash equilibrium always exists, and when the market is at Nash equilibrium, we have:*

$$P_i = \frac{1}{\sum_{j \in \mathcal{NR}(i)} \frac{l_{ij}}{2d_{ij}}} S_i, \quad \forall i \in \mathcal{N}. \tag{3}$$

where l_{ij} is the length of border line between i, j and d_{ij} is the distance between i, j.

Similarly to the 1D case, the factor $\gamma = \sum_{j \in \mathcal{NR}(i)} \frac{l_{ij}}{2d_{ij}}$ represents the competition intensity. For a company i, farther distance to competitors (bigger d_{ij}) can reduce the competition intensity, while longer contiguous border (bigger l_{ij}) increases it.

4. MARKET EQUILIBRIUM WHEN $Q = 1$

In this section, we discuss the situation when $q = 1$, i.e., when the market area has a linear relationship with the brand name. We show that the interesting "wipe out" phenomenon appears when $q > 0$.

The "wipe out" phenomenon substantially increases the difficulty in analyzing the problem, because in this case a company's market area can suddenly shrinks to zero after some threshold price. In this case, its neighbors' utility functions are not continuous. This is exactly the same problem as in the classic Hotelling model, where "undercut" destroys the continuity of the utility function, and therefore leads to the non-existence of equilibrium.

THEOREM 3. *Nash equilibrium always exists in the 1D market with $q = 1$.*

Note that proving the existence of equilibrium under the possibility of "wipe out" is highly non-trivial, since any "sudden death" company may lead to chain reaction of all companies' pricing strategies. In fact, we can show that for any company i, the necessary and sufficient condition of surviving in the market is $\beta < \frac{2d_i d_{i-1}}{d_i + d_{i-1}}$. In another world, companies can survive better with farther distances to neighbors (greater d_i, d_{i-1}) or in a market with less brand effect (smaller β). Moreover, we can also show results similar to Theorem 2, but due to the "wipe-out" phenomenon, companies' strategies will be more conservative, and equilibrium prices will be lower than those under $q = 0$.

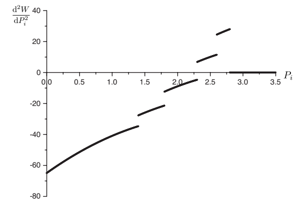

Figure 2: An example of second order derivative of the utility function. Each piece is a fragment from an increasing downward parabola.

5. CONCLUSION

In this paper, we study equilibrium properties based on a variant Hotelling model, considering brand name effect in the market by including a market area term into customers' aggregate price. We prove the existence of Nash equilibrium in single-feature and dual-feature market, and also derive explicit characterizations of equilibrium prices and market areas. Our results reconcile the common belief that company's pricing power is proportional to its market area over competition intensity, and offer insight into pricing under brand name effect and market positioning.

Specifically, our results offer the following insight: (i) When there is no brand effect or equivalent brand effect, i.e., β or q is zero, the equilibrium price of a company is proportional to its market area (market power) over the competition intensity with its neighbors (boundary over distance). (ii) When brand name has a positive effect in attracting customers, it is important to lower the price and seize more market area. (iii) New companies should try to avoid markets where the brand factor is large, and to avoid positioning at market points where competition is intense, because they can be "wiped out".

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