Optimal Smart Grid Tariffs

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***Abstract*—In this paper, we develop a low-complexity scheme OpTar for designing optimal power tariffs. OpTar provides an easy way for the utility companies to adjust their power prices, and allows the users to pre-plan their power consumption. OpTar is robust against system dynamics including time-varying power market conditions and user preferences. It can be implemented both with and without communication between the utility company and the consumers, and requires minimum additional hardware/software. We show that OpTar can significantly outperform the commonly used uniform pricing schemes. The pricing design approach developed in the paper can also be used in other systems with dynamic demand and non-storable resources.**

I. INTRODUCTION

Recently, there has been an increased effort in developing efficient demand response algorithms for the emerging smart grid [5]. Demand response techniques can not only reduce the peak load of the system, which improves the stability of the power grid, but also "shape" the demand according the available renewable energy to reduce the carbon emission of power generation.

In this paper, we consider the problem of designing optimal power tariff schemes for utility companies. Our objective is to develop implementable low-complexity tariff schemes that can guide the users to plan their loads at times that are favored by the system, and provide performance guarantees under user and system dynamics. Specifically, we consider a utility company supplying power to a set of customers. Everyday, the utility company decides the power tariff profile for the next day and announces it to the users ahead of time. At the same time, the utility company also decides how much power to procure for the next day, and spends the corresponding cost for procuring the power. After receiving the prices, each user greedily optimizes his power consumption plan for the next day, based on his preferences on power consumption. Then, on the next day, based on the supply and demand condition, the utility company tries to update the prices. The overall goal of the utility company is to find the optimal tariff profile such that the social welfare, i.e., the total user satisfaction minus the power cost, is maximized.

There have been many previous works on demand response that consider similar models. The work in [6] develops a TCP rate control like demand response algorithm. Work [16] models a static system and develops a real-time pricing algorithm. Works [4], [12], [11] develop two timescale power procurement and demand response algorithms that maximize the expected social welfare, under both time-independent and time-dependent demands. The work [10] develops a power procurement and demand response algorithm that maximizes the social welfare and guarantees the users' demands are met with finite delay. Work [19] considers the demand response problem using mechanism design. The work [17] also consider the problem from a game-theoretic perspective. The work [2] designs peak load pricing for systems with demand and supply uncertainties. Work [3] proposes using priority pricing and dynamic pricing to hedge the financial risk and secure electricity for the consumers. Outside the electricity network area, recent works [14] and [21] design *time-dependent-pricing* for data networks to alleviate congestion.

However, most of the previous works either (a) focus on a static system, or (b) consider real-time dynamic pricing and assume users are willing to partially adjust their loads based on the pricing signal, or (c) require the users to reveal private information to the utility company, or (d) do not apply to case when demands are correlated over time intervals. We focus instead on developing *optimal* tariff schemes that have very low complexity and are robust to system dynamics. In our design, the power prices are announced to the users ahead of time, and the users do not need to pass information to the utility company. Such a scheme not only greatly reduces the communication overheads between the utility company and the users, but also allows the consumers to pre-configure their power consumption over multiple time intervals. It also requires only that each user is able to solve some small planning problem with a few variables each day, and hence is immediately implementable in today's power grid. Our scheme is similar in nature to the Time-Varying Pricing by PG&E [20], with the philosophy that demand response schemes that are too complicated are hard to be widely adopted.

The paper is organized as follows. In Section II we state our system model. In Section III we present the algorithm design approach and the algorithm. We then analyze the algorithm in Section IV. Simulation results are presented in Section VI. We conclude the paper in Section VII.

II. SYSTEM MODEL

We consider a system where the utility company is supplying power to a set of N users, denoted by $\mathcal{N} = \{1, 2, ..., N\}$. Time is divided into periods of days, and each day is further divided into T equal size slots that correspond to, e.g., one-hour or 15-minute intervals. We denote the set of slots by $\mathcal{T} = \{1, 2, ..., T\}$. An example of the time slot structure is shown in Fig. 1.

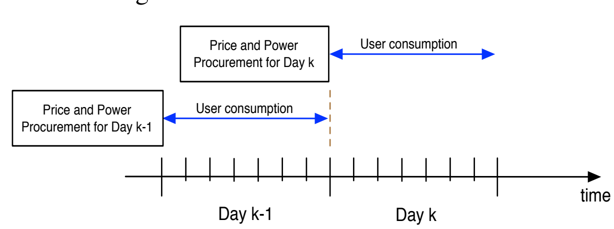

Fig. 1. The time slot model.

A. The power tariff and load planning process

We assume that one day before the day under consideration, the utility company selects a power tariff profile $\boldsymbol{p} = (p_1, ..., p_T)$ and announces it to the users. Based on the tariff, the utility company receives power demand for each time slot. On the other hand, in order to meet the power demands, the utility company also has to decide how much power to procure from the grid for the next day, and spends the corresponding costs.

To model the fact that the costs depend on the operating conditions of the power system, e.g., power market prices and available renewable energy, and can be time-varying we assume that there is a *power system condition* $m \in \mathcal{M} = \{m_1, ..., m_K\}$ that governs the power procurement costs. Here $\mathcal{M}$ is the set of possible system conditions, and is assumed to be a finite but arbitrarily large set. One example of the system condition m is:

$$m = (da_t, \mathbb{E}\big[rt_t\big], f_{\text{renewable}}(r_t), d_{\text{base},t}, t \in \mathcal{T}), \tag{1}$$

where da_t is the day-ahead power price for slot t on the next day, rt_t is the forecasted real-time power price for slot t on the next day, r_t is the amount of renewable energy available at slot t with a p.d.f. $f_{\text{renewable}}(r_t)$, and $d_{\text{base},t}$ is the *base load* at time t. Such a base load can represent some unchangeable load due to, e.g., long term contracted supply or *unplanned* user demand. We assume that the utility company can observe the power system condition when making decisions for the price and power procurement. This is not a restrictive assumption. In practice, the utility company can always access information such as the day-ahead prices and wind power forecast.

Under a system condition m, if the utility company decides to procure a total amount of power D at time t, it has to spend an *expected* cost of $C_t(D, m)$. We assume that $C_t(D, m)$ is a continuous function of D for all m and t. Below, for ease of presentation, we will state our results assuming that m is i.i.d. over every day. The results in this paper can be extended to the Markovian case using the technique developed in [9].

B. The user model

Upon receiving the prices, each user plans how to consume his load in different time slots. We denote $\boldsymbol{d}_n = (d_{n1}, ..., d_{nT})$ the consumption profile of user n, where d_{nt} denotes the load consumed at time t. In practice, the user behaviors are typically varying over time. To model this aspect, we assume that each user n has a *user condition* x_n which captures his constraints and preferences on how to consume his power. For example, the user condition can be the user's favorite and available time slots for doing his laundry. We assume that $x_n \in \mathcal{X}_n = \{x_{n1}, ..., x_{nJ_n}\}$, where $\mathcal{X}_n$ denotes the set of possible user conditions of user n. We similarly assume that $\mathcal{X}_n$ is a finite and time-invariant set, and present our results assuming that x_n is i.i.d. over each day. Each user can observe the value of x_n when planning their power consumption.

Under a user condition x_n, each user has a *feasible* power consumption set $\mathcal{D}_n(x_n)$, which specifies the constraints on the power needs of the user. For instance, one example of $\mathcal{D}_n(x_n)$ can be:

$$\mathcal{D}_n(x_n) = \{\boldsymbol{d}_n : d_{nt} \geq 0, \sum_t d_{nt} \leq d_n(x_n)\}, \tag{2}$$

where $d_n(x_n)$ is a constant that depends on x_n. Thus, the user has a maximum consumption level $d_n(x_n)$ and can consume his load in any slot. Another example of $\mathcal{D}_n(x_n)$ can be:

$$\mathcal{D}_n(x_n) = \{\boldsymbol{d}_n : d_{nt} \in \{0, 1\}, \sum_{t \neq 1} d_{nt} = 1\}.$$

In this case, the users wants to consume one unit of load, and has to consume the entire unit in a single slot other than slot 1. Other cases where the entries in the power profile may have time correlation can also easily be modeled by specifying $\mathcal{D}_n(x_n)$. We assume that each feasible set $\mathcal{D}_n(x_n)$ is a compact subset of $\mathbb{R}^T$ for all x_n. Thus, there exists a constant $d_{\max}$ such that $\sum_t d_{nt} \leq d_{\max}$ for all n and all x_n.

To model the users' satisfaction derived from his power consumption, we assume that each user receives a utility $U_n(\boldsymbol{d}_n, x_n)$ under the power consumption profile $\boldsymbol{d}_n$ and the user condition x_n. We assume $U_n(\cdot, x_n)$ is concave increasing and time-invariant. Note that $U_n(\cdot, x_n)$ can also represent the *expected* utility if the user has certain power demand that is random. Examples of the utility functions include:

- Linear utility: $U_n(\boldsymbol{d}_n, x_n) = \sum_{t=1}^T w_{nt}(x_n) d_{nt}$, where $w_{nt}(x_n) \geq 0$ represents the utility user n derives by consuming one unit power in time t.
- Concave utility: $U_n(\boldsymbol{d}_n, x_n) = \sum_{t=1}^T u_n(d_{nt}, x_n)$, where $u_n(d_{nt}, x_n)$ is a concave increasing function of d_{nt}.

Finally, we assume that each user is *selfish*. Thus, when planning the load allocation, each user will simply optimize the following problem to determine his own power profile:

$$\max: \ U_n(\boldsymbol{d}_n, x_n) - \sum_{t=1}^T p_t d_{nt}, \quad \text{s.t. } \boldsymbol{d}_n \in \mathcal{D}_n(x_n). \tag{3}$$

Since the set $\mathcal{D}_n(x_n)$ is compact and the function $U_n(\cdot, x_n)$ is continuous, the maximum is well defined.

C. The objectives

We assume that the utility company's goal is to design a power tariff $\boldsymbol{p}$ to maximize the *expected* social welfare, defined as follows:

$$W \triangleq \mathbb{E}\bigg[\sum_{n=1}^N U_n(\boldsymbol{d}_n(x_n), x_n) - \sum_{t=1}^T C_t(\sum_{n=1}^N d_{nt}(x_n), m)\bigg], \tag{4}$$

subject to the user's consumption profile being determined by (3). Here the expectation is taken over the power system condition m and the user condition x_n.

Below we use W^* to denote the maximum value of (4). Also, for notation simplicity, we denote $s = (m, x_n, n \in \mathcal{N})$ the aggregate *grid state*, and denote $\mathcal{S} = \mathcal{M} \times \prod_n \mathcal{X}_n$ its state space. We also compactly rewrite $\mathcal{D}_n(x_n)$, $U_n(\boldsymbol{d}_n(x_n), x_n)$ and $C_t(D, m)$ as $\mathcal{D}_n^s$, $U_n^s(\boldsymbol{d}_n^s)$ and $C_t^s(D)$, and use them interchangeably.

III. Tariff Design for Welfare Maximization

In this section, we study how to design the optimal tariffs. Our goal is to align the users' interests with the system's preference, i.e., motivate the users to consume their load at time slots that are favored by the system, e.g., when the user utility is high or when the cost is low. We also want our algorithm to be *fully distributed* and require minimum information exchange between the users and the utility company. However, directly solving the problem (4) can be challenging because (a) the cost $C_t^s(\sum_{n=1}^N d_{nt}^s)$ couples all the user demands, (b) the utility company may not have knowledge of the users' utility functions $\{U_n^s(\cdot)\}$ and their feasible power sets $\{\mathcal{D}_n^s\}$, and it can be too costly to obtain all these information. Indeed, due to the randomness in the grid state, it is very difficult to solve the problem (4) exactly unless the users and the utility company collaborate and schedule everything before hand.

To carry out our tariff design, we introduce a constant vector $\boldsymbol{\gamma} = (\gamma_1, ..., \gamma_T)$ and auxiliary variables $\{\boldsymbol{q}^s = (q_1^s, ..., q_T^s), \forall\, s \in \mathcal{S}\}$, and transform the welfare maximization problem (4) into the following *approximate* problem:

$$W_{\boldsymbol{\gamma}}^* \triangleq \max: \;\; \mathbb{E}\bigg[\sum_{n=1}^N U_n^s(\boldsymbol{d}_n^s) - \sum_{t=1}^T C_t^s(q_t^s)\bigg], \tag{5}$$

$$\text{s.t.} \;\; \mathbb{E}\bigg[\sum_{n=1}^N d_{nt}^s\bigg] \leq \mathbb{E}\bigg[\gamma_t q_t^s\bigg], \;\forall\, t, \tag{6}$$

$$\boldsymbol{d}_n^s \in \mathcal{D}_n^s, \;\forall\, n, s, \;\; 0 \leq q_t^s \leq q_{\max}, \;\forall\, t.$$

Here q_t^s can be viewed as the power procured by the utility company under the grid condition s (Below we call the vector $\boldsymbol{q}^s = (q_1^s, ..., q_T^s)$ the *supply profile* for state s). $q_{\max} \triangleq N d_{\max}$ is an upper bound on the total power demand. The γ_t constants measure how the utility company concerns about under-provisioning the power supply (to be further explained later in Section IV-B). A smaller γ_t value indicates that the utility company is more unwilling to have under-provision. We note that the introduction of $\boldsymbol{q}^s$ is important, because *it decouples the user actions and enables the development of low-complexity and distributed pricing scheme.* Note that when there is only one grid state, i.e., the system is static, and when $\gamma_t = 1$, the two problems (4) and (5) are equivalent. In this case, $W_{\mathbf{1}}^* = W^*$. In general, since (6) is a weaker constraint than $\sum_{n=1}^N d_{nt}^s \leq q_t^s$, we have $W_{\mathbf{1}}^* \geq W^*$.

Finally, if $U_n^s(\cdot)$ is strictly concave and $C_t^s(\cdot)$ is strictly convex, the objective function (5) is a strictly concave function. In the case where each $\mathcal{D}_n^s$ is also a convex set, there will be a unique optimal solution for problem (5).

A. The Optimal Tariff Algorithm (OpTar)

We now design our algorithm for finding the optimal tariff. The idea is to first obtain the dual problem of (5). Then, solve the dual problem using an *incremental* subgradient method [1], by using the Lagrange multiplier as the price profile. Specifically, the dual problem of (5) is given by:

$$\min: g_{\boldsymbol{\gamma}}(\boldsymbol{\lambda}), \quad \text{s.t. } \boldsymbol{\lambda} \succeq \mathbf{0}, \tag{7}$$

where $\boldsymbol{\lambda} = (\lambda_1, ..., \lambda_T)$ is the Lagrange multiplier, and $g_{\boldsymbol{\gamma}}(\boldsymbol{\lambda})$ is the dual function defined as:

$$g_{\boldsymbol{\gamma}}(\boldsymbol{\lambda}) \triangleq \sup_{\boldsymbol{d}_n^s \in \mathcal{D}_n^s, q_t^s \geq 0} \mathbb{E}\bigg[\sum_{n=1}^N U_n^s(\boldsymbol{d}_n^s) - \sum_{t=1}^T C_t^s(q_t^s) \tag{8}$$

$$- \sum_{t=1}^T \lambda_t \big[\sum_{n=1}^N d_{nt}^s - \gamma_t q_t^s\big]\bigg].$$

It is well known that the dual function $g_{\boldsymbol{\gamma}}(\boldsymbol{\lambda})$ is always convex even if both the objective function and the feasible consumption sets $\mathcal{D}_n^s$ are non-convex [1]. Indeed, $g_{\boldsymbol{\gamma}}(\boldsymbol{\lambda})$ is the supremum of a set of linear functions. Below, we use $\boldsymbol{\lambda}^*$ to denote a maximizer of $g_{\boldsymbol{\gamma}}(\boldsymbol{\lambda})$, and use $g_{\boldsymbol{\gamma}}^* = g_{\boldsymbol{\gamma}}(\boldsymbol{\lambda}^*)$ to denote the maximum value of the dual problem. Note from weak duality that we always have $g_{\boldsymbol{\gamma}}^* \geq W_{\boldsymbol{\gamma}}^*$.

Based on (7) and (8), we now develop our algorithm for finding the optimal power tariff. Recall that the $\boldsymbol{\gamma}$ parameter in the algorithm is a fixed vector chosen by the utility company to account for power under-provision, and $s = (m, x_n, n \in \mathcal{N})$ is the aggregate grid state. A pictorial illustration of our scheme is given in Fig. 2.

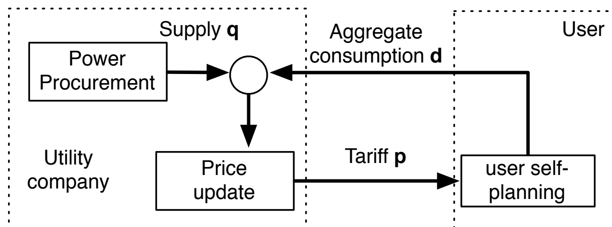

Fig. 2. The Optimal Tariff (OpTar) algorithm. The utility company chooses the prices and the power procurement, and broadcasts the prices to the users. The users then greedily plan their consumptions. The utility company observes the *aggregate* demand in every time slot and updates the prices.

<u>Optimal Tariff (OpTar):</u>

- Initialize $\boldsymbol{\gamma}$, $\boldsymbol{\lambda}(0) = \mathbf{0}$ for all $t = 1, ..., T$. [1]
- At iteration $k = 0, 1, 2, ...$, do:
  - (**Tariff advertising and power procurement**) The utility company broadcasts $\boldsymbol{p} = \boldsymbol{\lambda}(k)$ to the users, i.e., using $\boldsymbol{\lambda}(k)$ as the tariff profile. Then, the utility company observes the current power system condition m and computes the supply profile $\boldsymbol{q}(k)$ by solving:

$$\min: \sum_{t=1}^T [C_t(q_t, m) - \gamma_t \lambda_t^k q_t], \;\; \text{s.t. } 0 \leq q_t \leq q_{\max} \;\; \forall\, t. \tag{9}$$

  - (**User consumption planning**) Each user n observes the current user condition x_n and solves problem (3) to find the optimal load profile $\boldsymbol{d}_n(k)$, i.e.,

$$\max: U_n(\boldsymbol{d}_n, x_n) - \sum_{t=1}^T \lambda_t^k d_{nt}, \quad \text{s.t. } \boldsymbol{d}_n \in \mathcal{D}_n(x_n).$$

  - (**Tariff update**) The utility company observes the *aggregate* demand in each time t, i.e., $\sum_n d_{nt}(k)$, and updates $\boldsymbol{\lambda}(k)$ by:

$$\lambda_t(k+1) = \big[\lambda_t(k) + \epsilon_k (\sum_{n=1}^N d_{nt}(k) - \gamma_t q_t(k))\big]^+. \tag{10}$$

    Here $\epsilon_k, k = 0, 1, ...$ is the step size.

We note that OpTar *does not require any prior statistical knowledge* of the system condition m and the user conditions

[1]In the case when the utility company has prior knowledge of the user behavior and the power system condition, the utility company can "jump start" the algorithm by pre-computing a *near-equilibrium* prices.

$\{x_n\}_{n\in\mathcal{N}}$. This is particularly useful when the system dynamics are volatile and the distributions are hard to known exactly. Also, the utility company *does not need to know any user information.* This greatly reduces the communication overhead.

OpTar also has the useful feature that it can be implemented in both scenarios when the users negotiate with the utility company (the **negotiation** mode) and when they simply respond to the prices every day (the **daily-adjustment** mode):

- In the **negotiation mode**, OpTar can be implemented to exactly solve the welfare maximization problem (5) for any given power system and user conditions via message passing between the utility company and the users. In this case, each iteration is a single round of message passing and can be very fast. This implementation is more desirable when the communication cost is low.
- In the **daily-adjustment mode**, the users do not communicate with the utility company and only respond to the given tariff everyday. Thus, there will not be any message exchange between the users and the utility company. In this case, each iteration corresponds to a day. This implementation is compatible with today's power system, where the utility company adapts its price profile based on the aggregate user demand and the power system condition.

As we will see later, when OpTar is implemented in the daily-adjustment mode, there can be times when the power is under-provisioned due to system dynamics and lack of exact user information. In such a case, additional cost can occur due to, e.g., purchasing additional power from the spot market, or curtailing users's demand [3] (This will be further discussed in Section IV-B). We also note that under OpTar, each user will perform a self-planning of his own load. This approach is different from the partial adjustment scheme used in [16] and [11]. Finally, OpTar can easily be implemented with the power tariffs being changed more infrequently, without losing its optimality. This makes the algorithm very suitable for power tariff design problems where the prices can only be changed very infrequently.

IV. Performance analysis of OpTar

In this section, we analyze the performance of OpTar. We first study its equilibrium behavior and show that the prices converge to an optimal value under OpTar with appropriate step size rules. Then, we study the performance of OpTar when it is implemented in the daily-adjustment mode. In this case, we show that OpTar also guarantees a close-to-optimal welfare performance.

A. Equilibrium analysis

We first study the equilibrium behavior of OpTar. The performance results are summarized in the following theorem. In the theorem, we denote $\boldsymbol{q}(\boldsymbol{\lambda})$ and $\boldsymbol{d}_n(\boldsymbol{\lambda})$ the primal solutions corresponding to a dual variable $\boldsymbol{\lambda}$.

Theorem *1:* (a) Suppose the feasible consumption sets $\{\mathcal{D}_n(x_n)\}$ are convex, the utility functions are strictly concave, and the cost functions are strictly convex. Then $g^*_{\gamma}=W^*_{\gamma}$, and $(\boldsymbol{q}(\boldsymbol{\lambda}^*), \boldsymbol{d}_n(\boldsymbol{\lambda}^*), n\in\mathcal{N})$ is a feasible optimal solution of (5).

(b) If the step sizes $\{\epsilon_k, k=0,1,2,...\}$ satisfy $\sum_k\epsilon_k=\infty$ and $\sum_k\epsilon_k^2<\infty$, then under OpTar, $\boldsymbol{\lambda}(k)\rightarrow\boldsymbol{\lambda}^*$ with probability 1, where $\boldsymbol{\lambda}^*$ is an optimal solution of (7). $\diamond$

Proof: See [1]. ■

We note an interesting property of the resulting tariff: *it aligns the interests of the users and that of the system.* To see this, assume that both the power system and user conditions are fixed for all time, and omit the indexes m and x_n. Then, from (6) and (9), we see that when the condition in (a) in Theorem 1 holds, at the optimal solution, we have:

$$q^*_t=\sum_n d^*_{nt},\quad \lambda^*_t=\frac{\partial C_t(q^*_t)}{\partial q_t}. \tag{11}$$

This implies that the optimal tariff profile $\boldsymbol{p}$ satisfies:

$$p^*_t=C'_t(\sum_n d^*_{nt}),\ \forall\ t. \tag{12}$$

That is, the price for each time slot t is equal to the marginal cost of procurement. Such a pricing scheme is known as the "externality" in network routing games, and has been shown to be able to align the user's interest with that of the society [13]. Specifically, suppose that $\mathcal{D}_n=\{\sum_t d_{nt}\leq d_n, d_{nt}\geq 0\}$ and $U_n(\boldsymbol{d}_n)=\sum_{t=1}^T u_{nt}(d_{nt})$. Then, user n's optimization problem becomes:

$$\begin{aligned}\max:&\quad \sum_{t=1}^T u_{nt}(d_{nt})-\sum_{t=1}^T p_t d_{nt},\\ \text{s.t.}&\quad \sum_t d_{nt}\leq d_n, d_{nt}\geq 0.\end{aligned}$$

Associating with the first constraint a Lagrange multiplier η_n, the KKT condition [1] states that at the user's optimal consumption profile, we have:

$$u'_{nt}(d_{nt})-p_t=\eta_n,\qquad \text{if } d_{nt}>0, \tag{13}$$

$$u'_{nt}(d_{nt})-p_t\leq\eta_n,\qquad \text{if } d_{nt}=0. \tag{14}$$

Then, the welfare optimization problem is given by:

$$\begin{aligned}\max:&\quad \sum_{n=1}^N\sum_{t=1}^T u_{nt}(d_{nt})-\sum_{t=1}^T C_t(\sum_{n=1}^N d_{nt}),\\ \text{s.t.}&\quad \sum_t d_{nt}\leq d_n,\forall\, n, d_{nt}\geq 0.\end{aligned} \tag{15}$$

Similarly assigning Lagrange multipliers β_n to the first set of constraints, the KKT condition states that:

$$u'_{nt}(d_{nt})-C'_t(\sum_{n=1}^N d_{nt})=\beta_n,\qquad \text{if } d_{nt}>0, \tag{16}$$

$$u'_{nt}(d_{nt})-C'_t(\sum_{n=1}^N d_{nt})\leq\beta_n,\qquad \text{if } d_{nt}=0. \tag{17}$$

Thus, the social optimal condition and the selfish optimization problem are identical if we set $p_t=C'_t(\sum_{n=1}^N d_{nt})$.

B. Dynamic analysis

In this section, we study the performance of OpTar when it is implemented in the daily-adjustment mode. We note that in this case, the actual utility-user system evolution indeed simulates the dynamics of OpTar. Therefore, it is more meaningful to study the *average* performance of the algorithm, i.e., the average welfare achieved over time.

Different from the equilibrium analysis, here we must take into account the additional cost/saving due to the mismatch between the power supply and demand. To do so, we first recall that in the daily-adjustment implementation, each iteration corresponds to a day. Thus, if in a slot t, the aggregate demand is larger than the procured power, the utility company pays an extra cost $\alpha_{bt}(k)$ per unit unmet demand. Such a cost can be due to, e.g., curtailing the users' demand and compensating the users with certain prices [3].

On the other hand, if the procured power is more than the aggregate demand, we assume that the utility company can sell the excess power at a unit price $\alpha_{st}(k)$. For example, the utility company can sell the excess power back to the grid, or it can introduce a second market and sell the excess power to opportunistic users seeking for interruptible powers at low prices [8]. We assume that for each $t \in \mathcal{T}$, the vector $(\alpha_{bt}(k), \alpha_{st}(k))$ is i.i.d. drawn from some finite set Γ_t, which contains the constraint $0 \leq \alpha_{st}(k) \leq \alpha_{bt}(k) \leq \alpha_{\max}$. Also, $\alpha_{bt}(k)$ and $\alpha_{st}(k)$ satisfy: [2]

$$\mathbb{E}\big[\alpha_{bt}(k)\big] = \alpha_t, \quad \mathbb{E}\big[\alpha_{st}(k)\big] = \theta_t\alpha_t. \tag{18}$$

Here $\theta_t \leq 1, \forall t$ is to model the fact that under-provision incurs more costs to the utility company. We assume that $\alpha_{st}(k)$ is *uncorrelated* with $q_t(k)$, i.e., $\mathbb{E}\big[q_t(k)\alpha_{st}(k)\big] = \mathbb{E}\big[q_t(k)\big]\mathbb{E}\big[\alpha_{st}(k)\big]$. This can be the case, e.g., when $\alpha_{st}(k)$ is determined by the secondary market mentioned above.

To quantify the effect of the mismatch between demand and supply on the achieved welfare, we define $H_t(k)$ to be the total extra payment the utility company pays due to the mismatches between the supply and demand *up to day* k. $H_t(k)$ thus evolves according to:

$$\begin{aligned} H_t(k+1) = H_t(k) &+ [\sum_n d_{nt}(k) - q_t(k)]^+\alpha_{bt}(k) \\ &- [q_t(k) - \sum_n d_{nt}(k)]^+\alpha_{st}(k). \end{aligned} \tag{19}$$

The *actual* time average social welfare is then defined as:

$$W_{\text{av}}^{\text{actual}} \triangleq \liminf_{K\to\infty} \frac{1}{K}\left[\sum_{k=0}^{K-1}\mathbb{E}\big[W(k)\big] - \sum_t \mathbb{E}\big[H_t(K)\big]\right], \tag{20}$$

where $W(k)$ is the welfare achieved on day k *assuming no mismatch cost*, i.e.,

$$W(k) \triangleq \sum_{n=1}^{N} U_n^{s(k)}(\boldsymbol{d}_n(k)) - \sum_{t=1}^{T} C_t^{s(k)}(q_t(k)). \tag{21}$$

Here $s(k) = (m(k), x_n(k), n \in \mathcal{N})$ is the grid state of day k. We present the performance results of OpTar in the following theorem. In the theorem, we assume that the utility company only makes its decisions based on $\boldsymbol{\theta} = (\theta_1, ..., \theta_T)$, but not other knowledge of $\alpha_{bt}(k)$ and $\alpha_{st}(k)$.

Theorem 2: Suppose (a) $\boldsymbol{\gamma} = \boldsymbol{\theta}$, $\epsilon_k = \epsilon$ for all k, (b) each user n can solve the planning problem (3) to within σ_n of the maximum every day, and the utility company solves (9) to within σ_s, and (c) there exist a set of solutions $\{\boldsymbol{d}_n^s(l), n \in \mathcal{N}, q_t^s, t = 1, ..., T\}_{l=1}^{\infty}$ to (5), a set of weights $\{a^s(l)\}_{l=1}^{\infty}$ with $a^s(l) \geq 0, \sum_l a^s(l) = 1$, and a constant $\xi > 0$, such that:

$$\mathbb{E}\left[\sum_l a^s(l)\sum_n d_{nt}^s(l) - \theta_t q_t^s\right] \leq -\xi, \ \forall\ t. \tag{22}$$

Then, the *actual* average social welfare achieved by OpTar, denoted by $W_{\text{av}}^{\text{OpTar}}$ satisfies:

$$W_{\text{av}}^{\text{OpTar}} \geq W_{\boldsymbol{\theta}}^{*} - \epsilon TB - \sigma. \tag{23}$$

where $\sigma \triangleq \sum_n \sigma_n + \sigma_s$. $\diamond$

Proof: See Appendix A. ■

Note that (23) quantifies the effect of the difference between the buying and selling prices on the performance of OpTar. It is worth noting that Condition (b) is very useful, and can be used to model two practical scenarios. The first scenario is when the users know their utility functions exactly, but are not able to compute the optimal planning. The second scenario is when the users *do not* know the true utility function. In both cases, we see that Theorem 2 can be used to study the system performance. Similarly, the utility company is also allowed to make inexact decisions or to have inexact estimations of the power procurement cost. Theorem 2 thus shows that, *OpTar is robust against such inaccuracy and errors* in the decision making process.

V. Implementation

The OpTar algorithm can easily be implemented with today's technology. Below we outline a practical way to implement the algorithm.

First, the utility company's steps are easy to carry out. Advertising the power tariff only requires posting it onto a designated website or broadcasting it to the users either via cellular network to the smart meters or cell phones. Procuring power can also be done quickly by solving (9), which is an optimization problem of one variable. Updating the tariff is also very easy. The utility company only has to observe the *aggregate* user consumption over each time slot and update the tariff according to (10).

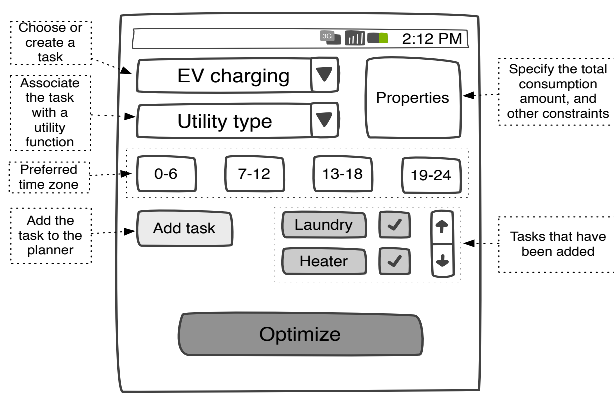

Fig. 3. An example smartphone application design. In the application, the user inputs (1) the name of the task, (2) the properties, e.g., the total amount of work, maximum and minimum charging rates, time-dependency across slots, (3) the utility function associated with the task, and (4) the preferred time to perform the task.

Second, the users' step is also easy to implement. We only need a simple software application for smartphones, tablets,

[2]Here we assume that $\alpha_{bt}(k)$ and $\alpha_{st}(k)$ are homogeneous across time. Our results can also be extended to the heterogeneous case where different days have different $\alpha_{bt}(k)$ and $\alpha_{st}(k)$ statistics.

or personal computer to carry out the optimization. Fig. 3 shows one possible design for a smartphone application. In the application, we can pre-specify a set of representing utility functions. Then, each user associates each action with a utility function, and specifies the constraints on the consumption and the preferred time slots. The application can then compute a schedule for the user and directly broadcast the schedule to the smart appliances via WiFi. Such a planning problem will typically involve no more than 100 variables with a set of linear constraints, and can be solved in a few seconds. To facilitate the computation, one can also assign a set of dedicated servers to solve these planning problems for the smartphone applications.

VI. SIMULATION

In this section, we present simulation results of our algorithm. We assume $T = 24$ and each slot is one hour long. We assume there are $N = 50$ users.

We assume there are three representing type of users. Each type j is associated with a vector $\boldsymbol{w}(j) = (w_t(j), t \in \mathcal{T})$, which specifies the unit consumption utilities obtained in different slots. We see that type 1 users prefer to consume loads in slots in $[9, 17]$; whereas type 2 users prefer slots in $[13, 21]$; while the weights for type 3 users are the same for all time. [3]

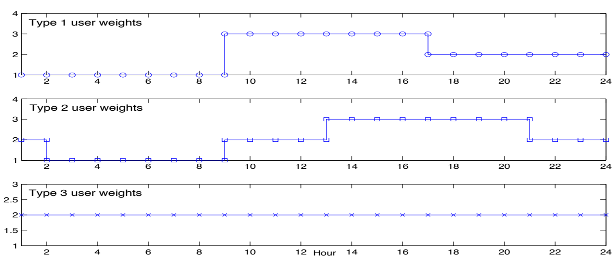

Fig. 4. Weights on time for three different types of users.

Each user's condition x_n indicates which type the user belongs to, i.e., $x_n \in \{1, 2, 3\}$. We assume each user chooses its condition i.i.d. every day with distribution $\{0.2, 0.7, 0.1\}$. So 70% of the users follow the aggregate residential consumption pattern. For a type x_n user, his utility from consuming power is given by:

$$U_n(\boldsymbol{d}_n, x_n) = 0.4 \sum_t \log(1 + w_t(x_n) d_{nt}). \tag{24}$$

Further, a type j user's constraint set is:

$$\mathcal{D}_n(x_n) = \{d_{nt} \geq 0, \sum_t d_{nt} \leq d_n(x_n)\}, \tag{25}$$

where $d_n(1) = 1$, $d_n(2) = 1.5$, and $d_n(3) = 2$. We also match the user demand unit to that of the power procurement by assuming that the users' consumption unit is $1/5$ of the power procurement unit. [4] For the power procurement cost, assume that the cost function for each time t is of the form:

$$C_t(D, m) = b_m D^2 + c_t D. \tag{26}$$

[3] The weights for Type 2 users are chosen to mimic the residential power consumption pattern found in [15], where the usage is higher from afternoon to night times.

[4] This is to capture the fact that in practice, users measure consumption in kWh whereas utility companies measure power procurement in units of MWh.

Here c_t is the cost coefficient and is given by:

$$c_t = \begin{cases} 0.5 \text{ (0.8 in 2nd half)} & t \in [1, 8], \\ 1.5 \text{ (2.5 in 2nd half)} & t \in [9, 18], \\ 1 \text{ (1.2 in 2nd half)} & t \in [19, T]. \end{cases} \tag{27}$$

To study how OpTar adapts to drastic environment changes, we change the value of the c_t values in the second half of the simulation. The values of c_t of the second half are given in the parenthesis. The b_m parameter represents the system state and is assume to take values 0.8 or 1.2 according to the Markov chain shown in Fig. 5. Finally, we assume that $\alpha_{bt}(k) = 3$ and $\alpha_{st}(k) = 2.7$ for all t. Thus, $\theta_t = 0.9$ for all t.

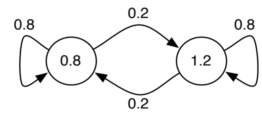

Fig. 5. The power system condition indicator b_m. The steady state probability is $0.5, 0.5$.

We run the simulation for 5000 iterations with a fixed step size $\epsilon_k = 0.01$ with CVX [7]. The change of cost coefficients takes place on the 2500-th iteration. Fig. 6 shows the instant welfare and running average welfare achieved by OpTar. We see that OpTar is able to quickly adapt to the change of coefficients. The reason for the fluctuation is due to the time-varying power system state b_m and the mismatch costs.

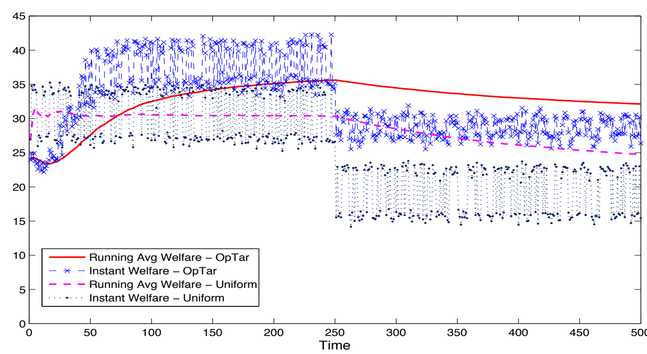

Fig. 6. The achieved social welfare under OpTar and Uniform with price 1 for all slots (sampled every 10 iterations). The results show that OpTar automatically adapts to the changing system and outperforms Uniform.

Fig. 6, we compare our scheme with a Uniform pricing scheme where the prices are the same for all slots. Such a scheme is very common in today's power systems. In this case, we assume that the utility company knows the users' utility function and constraint sets, as well as the user type distribution. The utility company first computes the consumption profile for each user type under the chosen price, and uses the user type distribution to estimate the expected total load in each slot. Then, it procures an amount of power which is equal to that expected load. For fair comparison, we verified via simulations that the price profile $p_t = 1$ for all t is optimal in the Uniform case. We can easily see from Fig. 6 that OpTar outperforms Uniform by more than $17\% - 28\%$ on average. Fig. 7 also shows a sampled aggregate demand in every slot. It shows that the demand under OpTar is more evenly spread out among slots compared to Uniform. This is indeed why OpTar is able to perform better than Uniform.

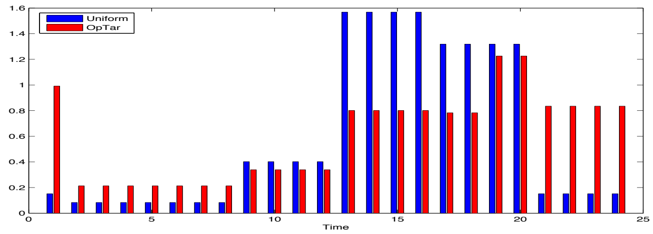

Fig. 7. A sample demand distribution under both OpTar and Uniform. It can be observed that OpTar leads to a better load balancing.

Fig. 8 also shows the power prices under OpTar. We see that the prices converge to the near-optimal values quickly and are able to adapt to the change of power system condition. The bottom plot also shows how the price for slot 1 changes with the system state. It can be seen that when the cost is high, the price will increase; whereas if the cost is low, the price will start to decrease. This shows that OpTar reacts quickly to the power cost change.

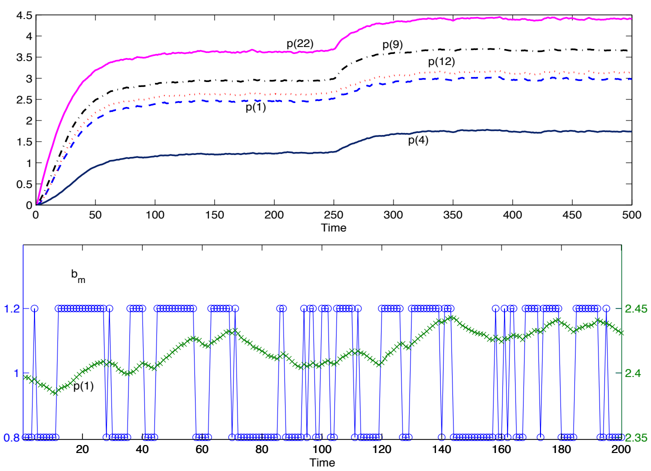

Fig. 8. Top: Selected prices dynamics under OpTar (sampled every 10 iterations). Bottom: The dynamics of the system state and the price for slot 1 during time $1000-1200$.

Finally, we look at the average utility obtained by each type of users. Table I shows that under OpTar, users of each type achieve a utility that is almost as good as that under Uniform (For the majority of the users, i.e., type 1 and 2, the utility under OpTar is more than 95% of that under Uniform). However, this 5% user satisfaction loss can improve the aggregate social welfare by up to 28%.

TABLE I
AVERAGE UTILITY FOR DIFFERENT TYPES OF USERS.

	Type 1 User	Type 2 User	Type 3 User
OpTar	0.9892	1.3557	1.2873
Uniform	1.0191	1.4295	1.4798
Ratio	0.97	0.95	0.87

VII. Conclusion

In this paper, we develop a low-complexity power tariff design scheme OpTar. OpTar requires minimum information of the system dynamics and can easily be implemented in today's power system. OpTar not only provides an easy way for the utility company to update the power prices but also allows the users to pre-plan their loads accordingly. We show that OpTar achieves a near-optimal performance, and significantly improves the social welfare upon uniform pricing schemes.

Appendix A - Proof of Theorem 2

In this section, we prove Theorem 2. Note that OpTar is implemented in the daily-adjustment mode. Also, $\epsilon_k=\epsilon$ for all k and $s(k)=(m(k),x_n(k),n\in\mathcal{N})$ is the grid state on day k.

Proof: (Theorem 2) We first study the social welfare *without considering the mismatch cost.* Squaring both sides of (10) with $\gamma_t=\theta_t$ and using the facts that $(\sum_n d_{nt})^2\leq N^2d_{\max}^2$ and $q_t^2\leq q_{\max}^2$, we have:

$$\lambda_t(k+1)^2\leq\lambda_t(k)^2+2\epsilon^2(N^2d_{\max}^2+\theta_t^2q_{\max}^2)\quad(28)$$
$$-2\epsilon\lambda_t(k)\big[\theta_tq_t(k)-\sum_n d_{nt}(k)\big].$$

Denote $B\triangleq(N^2d_{\max}^2+\theta_t^2q_{\max}^2)$. Summing the above over all $t\in\{1,...,T\}$ and multiplying by $\frac{1}{2\epsilon}$, we get:

$$\frac{1}{2\epsilon}\sum_t\lambda_t(k+1)^2\leq\frac{1}{2\epsilon}\sum_t\lambda_t(k)^2+\epsilon BT\quad(29)$$
$$-\sum_t\lambda_t(k)\big[\theta_tq_t(k)-\sum_n d_{nt}(k)\big].$$

Now define a Lyapunov function $\Delta(k)$ to be:

$$\Delta(k)\triangleq\frac{1}{2\epsilon}\mathbb{E}\big[\sum_t\lambda_t(k+1)^2-\sum_t\lambda_t(k)^2\mid\boldsymbol{\lambda}(k)\big].\quad(30)$$

By taking expectations over both sides of (29) conditioning on $\boldsymbol{\lambda}(k)$, and using (30), we get:

$$\Delta(k)\leq\epsilon BT-\mathbb{E}\bigg[\sum_t\lambda_t(k)\big[\theta_tq_t(k)-\sum_n d_{nt}(k)\big]\mid\boldsymbol{\lambda}(k)\bigg].$$

Subtracting from both sides the term:

$$\mathbb{E}\big[W(k)\mid\boldsymbol{\lambda}(k)\big]$$
$$\triangleq\mathbb{E}\bigg[\sum_{n=1}^N U_n^{x_n(k)}(\boldsymbol{d}_n(k))-\sum_{t=1}^T C_t^{m(k)}(q_t(k))\mid\boldsymbol{\lambda}(k)\bigg],$$

and rearranging the terms, we get:

$$\Delta(k)-\mathbb{E}\big[W(k)\mid\boldsymbol{\lambda}(k)\big]\leq\epsilon BT\quad(31)$$
$$-\mathbb{E}\bigg[\sum_{n=1}^N\big[U_n^{x_n(k)}(\boldsymbol{d}_n(k))-\sum_t\lambda_t(k)d_{nt}(k)\big]\mid\boldsymbol{\lambda}(k)\bigg]$$
$$+\mathbb{E}\bigg[\sum_{t=1}^T\big[C_t^{m(k)}(q_t(k))-\theta_t\lambda_t(k)q_t(k)\big]\mid\boldsymbol{\lambda}(k)\bigg].$$

Here the expectations are taken over the market condition $m(k)$ and the user conditions $x_n(k)$. We now note that OpTar *approximately* maximize the last expectation: First, it minimizes the last expectation for every $m(k)$ to within σ_s. Second, each user maximizes the corresponding term in the second to last term to within σ_n, i.e.,

$$U_n^{x_n(k)}(\boldsymbol{d}_n(k))-\sum_t\lambda_t(k)d_{nt}(k)\quad(32)$$
$$\geq\max_{\boldsymbol{d}_n\in\mathcal{D}_n(x_n(k))}\bigg[U_n^{x_n(k)}(\boldsymbol{d}_n)-\sum_t\lambda_t(k)d_{nt}\bigg]-\sigma_n.$$

Using the definition of the dual function in (8), we see that:

$$\Delta(k) - \mathbb{E}\big[W^{\mathsf{OpTar}}(k) \mid \boldsymbol{\lambda}(k)\big] \leq \epsilon BT + \sum_{n\in\mathcal{N}} \sigma_n + \sigma_s - g_{\boldsymbol{\theta}}(\boldsymbol{\lambda}(k)) \leq \epsilon BT + \sum_{n\in\mathcal{N}} \sigma_n + \sigma_s - g^*_{\boldsymbol{\theta}}.$$

The second step follows since $g^*_{\boldsymbol{\theta}}$ is the minimum value of $g_{\boldsymbol{\theta}}(\boldsymbol{\lambda})$. Denote $\sigma \triangleq \sum_{n\in\mathcal{N}} \sigma_n + \sigma_s$. By taking expectations over $\boldsymbol{\lambda}(k)$ and summing the above over $k = 0, 1, ..., K-1$, we obtain:

$$Kg^*_{\boldsymbol{\theta}} - K(\epsilon BT + \sigma) \leq \sum_{k=0}^{K-1} \mathbb{E}\big[W^{\mathsf{OpTar}}(k)\big] + \mathbb{E}\big[\boldsymbol{\lambda}(0)\big]/2\epsilon.$$

Dividing both sides by K and taking the $\liminf$ as $K \to \infty$, we get:

$$\liminf_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}\big[W^{\mathsf{OpTar}}(k)\big] \geq g^*_{\boldsymbol{\theta}} - (\epsilon TB + \sigma) \geq W^*_{\boldsymbol{\theta}} - (\epsilon TB + \sigma). \quad (33)$$

The last step follows by weak duality. This shows that *without the mismatch costs*, the social welfare achieved by OpTar is close-to-optimal.

To complete the proof, we only need to show that the average extra expenditure due to mismatches is negligible, i.e.,

$$\limsup_{K\to\infty} \frac{1}{K} \sum_t \mathbb{E}\big[H_t(K)\big] \leq 0. \quad (34)$$

Below, we first prove that (34) holds for $\boldsymbol{\lambda}(k)$. Then, we use the results for $\boldsymbol{\lambda}$ to prove (34) for $\boldsymbol{H}(k)$. To do so, note that by plugging the feasible solutions in (22) into (31), we get:

$$\Delta(k) - \mathbb{E}\big[W(k) \mid \boldsymbol{\lambda}(k)\big] \leq \epsilon BT - \xi \sum_t \lambda_t(k) - \mathbb{E}\Big[\sum_{n=1}^{N} \big[U_n^{s(k)}(\boldsymbol{d}_n(k)) - \sum_{t=1}^{T} C_t^{s(k)}(q_t(k)) \mid \boldsymbol{\lambda}(k)\Big]. \quad (35)$$

Now define

$$\delta_{\max} \triangleq \mathbb{E}\Big[\max_{\boldsymbol{d}_n^s\in\mathcal{D}_n^s, s} U_n^s(\boldsymbol{d}_n^s) + \max_{q_t^s, s} \sum_t C_t^s(q_t^s)\Big],$$

to be the maximum of the instant social welfare under any grid state, and rearrange the terms, we get:

$$\Delta(k) + \xi \sum_t \lambda_t(k) \leq \epsilon TB + \delta_{\max}. \quad (36)$$

Taking expectations on both sides over $\boldsymbol{\lambda}(k)$, summing over $k = 0, ..., K-1$, and rearranging the terms, we get:

$$\xi \sum_{k=0}^{K-1} \sum_t \mathbb{E}\big[\lambda_t(k)\big] \leq K\epsilon TB + K\delta_{\max} + \mathbb{E}\big[\sum_t \lambda_t(0)\big]/2\epsilon.$$

Dividing both sides by $K\xi$ and taking the $\liminf$ as $K \to \infty$ we get:

$$\liminf_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \sum_t \mathbb{E}\big[\lambda_t(k)\big] \leq \frac{\epsilon TB + \delta_{\max}}{\xi}. \quad (37)$$

Using a similar argument as in [18], this can be shown to imply:

$$\limsup_{K\to\infty} \frac{1}{K} \sum_t \mathbb{E}\big[\lambda_t(K)\big] = 0. \quad (38)$$

We now use (38) to prove (34). Consider the update rule of $\boldsymbol{\lambda}(k)$. Since $\gamma_t = \theta_t$, we see that:

$$\lambda_t(k+1) \geq \lambda_t(k) + \epsilon[\sum_n d_{nt}(k) - \theta_t q_t(k)].$$

This implies that:

$$\lambda_t(k+1) \geq \epsilon \sum_k [\sum_n d_{nt}(k) - \theta_t q_t(k)].$$

Using (38), we get that:

$$\limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}\big[\sum_n d_{nt}(k) - \theta_t q_t(k)\big] \leq 0. \quad (39)$$

Now from (19) we see that:

$$H_t(k+1) = \sum_k \Big([\sum_n d_{nt}(k) - q_t(k)]^+ \alpha_{bt}(k) - [q_t(k) - \sum_n d_{nt}(k)]^+ \alpha_{st}(k)\Big). \quad (40)$$

Define $I_{\mathcal{E}}$ the indicator function of an event $\mathcal{E}$. Also, denote $\mathcal{E}_b(k) = \{\sum_n d_{nt}(k) > q_t(k)\}$ the power "buying" event and $\mathcal{E}_s(k) = \{\sum_n d_{nt}(k) < q_t(k)\}$ the power "selling" event. We can rewrite the term inside the parenthesis of (40) as:

$$[\sum_n d_{nt}(k) - q_t(k)]^+ \alpha_{bt}(k) - [q_t(k) - \sum_n d_{nt}(k)]^+ \alpha_{st}(k)$$
$$= \sum_n d_{nt}(k) \Big[I_{\mathcal{E}_b(k)} \alpha_{bt}(k) + I_{\mathcal{E}_s(k)} \alpha_{st}(k)\Big] - q_t(k) \Big[I_{\mathcal{E}_b(k)} \alpha_{bt}(k) + I_{\mathcal{E}_s(k)} \alpha_{st}(k)\Big]$$
$$\leq \sum_n d_{nt}(k) \alpha_{bt}(k) - q_t(k) \alpha_{st}(k).$$

The last step uses the fact that $\alpha_{st}(k) \leq \alpha_{bt}(k)$ for all k, and $I_{\mathcal{E}_b(k)} I_{\mathcal{E}_s(k)} = 0$. Since $\alpha_{bt}(k)$ is only a function of $m(k)$, it is independent of $d_{nt}(k)$. Therefore, taking the expectations over both sides of the above equation and using the fact that $q_t(k)$ and $\alpha_{st}(k)$ are uncorrelated, we get:

$$\mathbb{E}\Big[[\sum_n d_{nt}(k) - q_t(k)]^+ \alpha_{bt}(k) - [q_t(k) - \sum_n d_{nt}(k)]^+ \alpha_{st}(k)\Big]$$
$$\leq \mathbb{E}\big[\sum_n d_{nt}(k)\big] \alpha_t - \mathbb{E}\big[q_t(k)\big] \theta_t \alpha_t.$$

Using this in (40), we see that:

$$\mathbb{E}\big[H_t(k+1)\big] \leq \alpha_t \sum_k \mathbb{E}\Big[\sum_n d_{nt}(k) - \theta_t q_t(k)\Big].$$

Therefore,

$$\limsup_{K\to\infty} \frac{1}{K} \mathbb{E}\big[H_t(K)\big] \quad (41)$$
$$\leq \limsup_{K\to\infty} \frac{\alpha_t}{K} \Big[\sum_{k=0}^{K-1} \mathbb{E}\Big[\sum_n d_{nt}(k) - \theta_t q_t(k)\Big]\Big]$$
$$\leq 0. \quad (42)$$

The last step follows from (39). This completes the proof of (34), and the proof of the theorem. ■

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